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Vehicle Guidance and Control Along Circular Trajectories

by

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Submitted in partial fulfillment
of the requirements for the degree of

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The problem of path keeping of marine vehicles along curved paths is considered. In particular, we are concerned with circular arcs that join two consecutive straight line segments. This provides smooth path changes between one segment and the next utilizing a purely geometric reference path construction. Pure pursuit guidance is coupled with orientation angle control law to ensure stability and accuracy. Sensitivity analysis with respect to inaccuracies in the knowledge of the vehicle hydrodynamic characteristics is performed. The results demonstrate the validity of this approach and offer a way to achieve accurate path keeping in confined spaces.

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I. INTRODUCTION

One of the most important functions of a marine vehicle is accurate path keeping through prescribed routes. Path accuracy is particularly important when the vehicle has to operate in confined waters. In the case of an unmanned vehicle, the commanded path is usually approximated by straight line segments between consecutive way points. This assumption works well if the desired route is a smooth path and can be approximated by a series of straight line segments. This has been studied extensively in the past. Lienard [1] designed a steering sliding mode control autopilot based on the line of sight, while Chism [2] studied a cross track error based control law. Hawkinson [3] extended the results to the multiple input case when bow and stern planes are independently controlled. Instead of using the cross track error directly in the control law, similar path keeping characteristics can be achieved by using a line of sight pursuit guidance scheme. This is accomplished by moving the vehicle in the direction of a moving way point which is located on the commanded straight line path [4]. Stability of this scheme depends on the control law gains and the look ahead distance between the vehicle and the moving way point [5]. The technique can be applied to combined steering and

diving response [6]. In this work, we extend the previous studies of pursuit guidance in the case of curved reference paths with piecewise constant curvature. This means that we adopt a purely geometrical construction of the reference path which is composed of a series of straight line segments and circular arcs. In Chapter II we present an overview of the equations of motion in the horizontal plane, the control and the guidance laws. We use the full set of maneuvering equations for simulation and a reduced model, Nomoto's first order model, for steering control design. This is because the vehicle parameters in the reduced order model can be estimated relatively accurately from first principles and experimental data [7]. In Chapter III the focus is on stability. We present two stability conditions, one based on the reduced model, and one based on the complete vehicle dynamics model. The sensitivity of the stability curves with respect to the vehicle hydrodynamic coefficients and control parameters is also studied in this chapter. In Chapter IV we present the development and simulation results of the guidance scheme for circular reference paths. All computations in this work are performed for the NPS AUV II vehicle for which a complete set of hydrodynamic coefficients and geometric properties is available [8]. Finally, we summarize the conclusions of this work and some recommendations for further research.

II. MATHEMATICAL FORMULATION

In this section the vehicle equations of motion for the horizontal plane (x,y), the control law and the guidance scheme are presented.

A. EQUATIONS OF MOTION

For the horizontal plane the mathematical model consists of the following simplified expressions for the nonlinear sway and yaw differential equations shown below :

$$m(\dot{v}+ur+x_G\dot{r})=Y \tag{2.1}$$

$$I_Z\dot{r}+mx_G(\dot{v}+ur)=N \tag{2.2}$$

Equations (2.1), (2.2) were derived from the general six degrees of freedom equations for a vehicle by setting all terms related to the vertical plane motion to be zero, assuming port/starboard symmetry. The equations for the sway force Y and yaw moment N are :

$$Y=Y_{\dot{r}}\dot{r}+(Y_{\dot{v}}\dot{v}+Y_r ur)+Y_v uv+Y_\delta u^2\delta$$

$$N=N_{\dot{r}}\dot{r}+(N_{\dot{v}}\dot{v}+N_r ur)+N_v uv+N_\delta u^2\delta$$

Finally the three kinematic equations of the inertial position rates and yaw rate are :

$$\dot{\psi}=r \tag{2.3}$$

$$\dot{x}=u\cos\psi-v\sin\psi \tag{2.4}$$

$$\dot{y}=u\sin\psi+v\cos\psi \tag{2.5}$$

B. CONTROL LAW

The coupled equations (2.1) and (2.2) after some algebra appear as shown :

$$\dot{v}=a_{11}uv+a_{12}ur+b_1u^2\delta \tag{2.6}$$

$$\dot{r}=a_{21}uv+a_{22}ur+b_2u^2\delta \tag{2.7}$$

where the coefficients a_{11}, a_{12}, a_{21}, a_{22}, b_1 and b_2 are :

$$D=(I_z-N_{\dot{r}})(m-Y_{\dot{v}})-(mx_G-Y_{\dot{r}})(mx_G-N_{\dot{v}})$$

$$a_{11}=\frac{1}{D}[(I_z-N_{\dot{r}})Y_v-(mx_G-Y_{\dot{r}})N_v]$$

$$a_{12}=\frac{1}{D}[(I_z-N_{\dot{r}})(-m+Y_r)-(mx_G-Y_{\dot{r}})(-mx_G+N_r)]$$

$$a_{21}=\frac{1}{D}(m-Y_{\dot{v}})N_v-(mx_G-N_{\dot{v}})Y_v]$$

$$a_{22}=\frac{1}{D}[(m-Y_{\dot{v}})(-mx_G+N_r)-(mx_G-N_{\dot{v}})(-m+Y_r)]$$

$$b_1=\frac{1}{D}\left[(I_z-N_{\dot r})\,Y_{\delta_s}-(mx_G-Y_{\dot r})\,N_{\delta_s}-(I_z-N_{\dot r})\,Y_{\delta_b}+(mx_G-Y_{\dot r})\,N_{\delta_b}\right]$$

$$b_2=\frac{1}{D}\left[(m-Y_{\dot v})\,N_{\delta_s}-(mx_G-N_{\dot v})\,Y_{\delta_s}-(m-Y_{\dot v})\,N_{\delta_b}+(mx_G-N_{\dot v})\,Y_{\delta_b}\right]$$

Nondimensionilizing the above equations in terms of the water density ρ the vehicle length L and the nominal forward speed U of the vehicle, results in the following :

$$\dot v=a_{11}v+a_{12}r+b_1\delta \tag{2.8}$$

$$\dot r=a_{21}v+a_{22}r+b_2\delta \tag{2.9}$$

From equations (2.8) and (2.9), the relationship of the vehicle turning rate r and the rudder angle δ appears through a second order transfer function :

$$\frac{r}{\delta}=\frac{b_2s+a_{21}b_1-a_{11}b_2}{s^2-(a_{22}+a_{11})\,s+a_{11}a_{22}-a_{12}a_{21}} \tag{2.10}$$

Performing a Taylor series expansion in the inverse of equation (2.10) and keeping the first two terms we arrive at a simplified first order transfer function :

$$\frac{r}{\delta}=\frac{c_3}{c_1+c_2s} \tag{2.11}$$

where :

$$c_1 = -(a_{11}a_{22} - a_{21}a_{12})(a_{21}b_1 - a_{11}b_2)$$

$$c_2 = (a_{11} + a_{22})(a_{21}b_1 - a_{11}b_2) + b_2(a_{11}a_{22} - a_{21}a_{12})$$

$$c_3 = -(a_{21}b_1 - a_{11}b_2)^2$$

Equation (2.11) can be also written as :

$$\dot{r} = ar + b\delta$$

where :

$$a = -\frac{c_1}{c_2} \qquad \text{and} \qquad b = \frac{c_3}{c_2}$$

In this way, the vehicle turning dynamics reduce to the approximate system of the following two equations :

$$\dot{\psi} = r \tag{2.12}$$

$$\dot{r} = ar + b\delta \tag{2.13}$$

The control law then will be based on ψ and r and this is going to be in the form :

$$\delta = k_1\psi + k_2 r \tag{2.14}$$

Solving for ψ from equations (2.12), (2.13) and (2.14) we end up with a second order differential equation as below :

$$\ddot{\psi} - (a+bk_2)\dot{\psi} - bk_1\psi = 0$$

In terms of the desired natural frequency ω_n and the damping ratio ζ the gains k_1 and k_2 are given by the following relations :

$$k_1 = -\frac{\omega_n^2}{b} \tag{2.15}$$

$$k_2 = -\frac{a+2\zeta\omega_n}{b} \tag{2.16}$$

Therefore, the feedback gains for the control law can be determined based on the desired values of ω_n and ζ.

C. GUIDANCE

The heading autopilot that was designed through the control law in the previous section is called upon now to provide vehicle path keeping through a series of way points in the horizontal plane and as we will see later accurate path keeping along circular paths. In order to achieve it and at the same time to keep the same heading autopilot design we have to use a suitable navigation scheme such as the line of sight guidance scheme (or the look ahead distance scheme). In the above scheme the autopilot attempts to position the

longitudinal axis of the vehicle towards a point D which is located ahead of the vehicle's nominal path at a fixed distance d as shown in Figure 1. This fixed distance d is the look ahead distance and the line of sight angle σ is defined as :

$$\tan\sigma = -\frac{y}{d} \tag{2.17}$$

For pure pursuit navigation the corresponding control law by modifying equation (2.14) becomes :

$$\delta = k_1(\psi - \psi_{com}) + k_2 r \tag{2.18}$$

where :

$$\psi_{com} = \sigma$$

By introducing equation (2.17) in the control law equation, the commanded vehicle heading angle is not constant any more but a function of the vehicle's position y as it can be seen from equation (2.5). Since the control law was based on equations (2.3), (2.6) and (2.7) without including equation (2.5) for lateral displacement, the selected gains k_1, k_2 do not guarantee stability any more and therefore conditions must be developed to guarantee stability and ensure satisfactory path keeping.

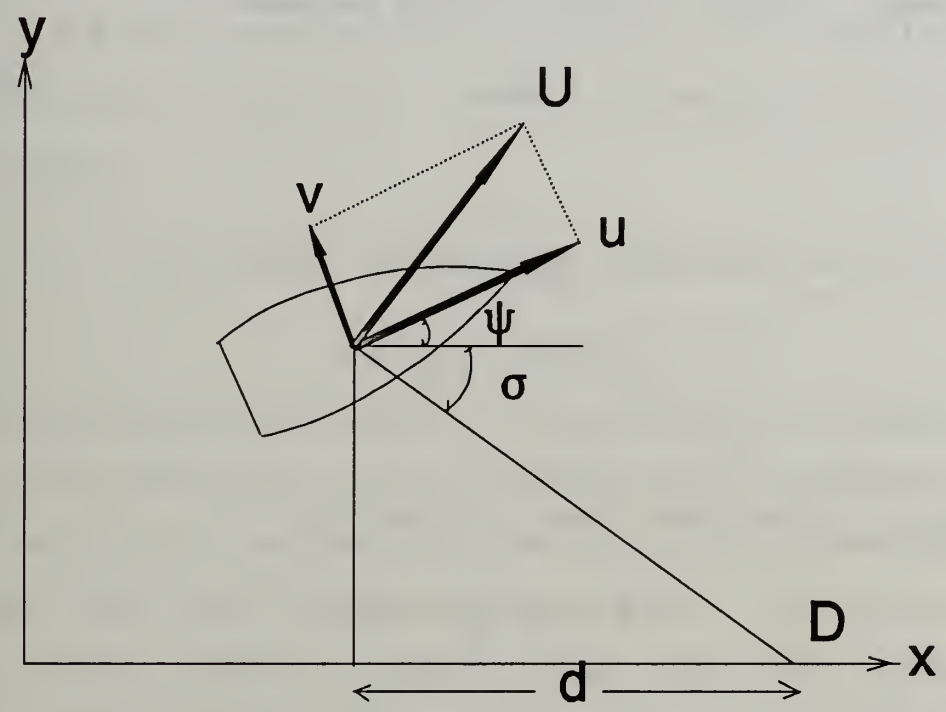

Figure 1. Geometry and Definitions of Symbols

III. STABILITY

A. APPROXIMATE CONDITION FOR STABILITY

So far the complete system is given by the approximate equations (2.12), (2.13), and the inertial position rate equation (2.5). In its linearized and dimensionless form for a very small lateral velocity v this becomes :

$$\dot{y}=\psi \tag{3.1}$$

The control law including the guidance scheme is :

$$\delta=k_1(\psi+\tan^{-1}\frac{y}{d})+k_2 r$$

which in its linearized form is given by :

$$\delta=k_1(\psi+\frac{y}{d})+k_2 r \tag{3.2}$$

Solving equations (2.12), (2.13), (3.1) and (3.2) for y we end up to with a third order differential equation as follows :

$$\dddot{y}-(a+bk_2)\ddot{y}-bk_1\dot{y}-\frac{bk_1}{d}=0 \tag{3.3}$$

Writing the above equation in the s-domain and substituting k_1 and k_2 from equations (2.15) and (2.16), equation (3.3) takes the following form :

$$s^3+2\zeta\omega_n s^2+\omega_n^2 s+\frac{\omega_n^2}{d}=0 \qquad (3.4)$$

The Routh criterion for stability, BC-AD > 0, where A, B, C and D are the coefficients of equation (3.4), applied to the above equation gives the following condition that must be satisfied :

$$d > \frac{1}{2\zeta\omega_n} \qquad (3.5)$$

If we have selected a natural frequency ω_n and a damping ratio ζ for the controller, it is possible from the above relation to have a quick estimate for the minimum look ahead distance d such that the system will be stable. In Figure 2 the approximate stability curve (dotted line) was plotted, the look ahead distance d versus the natural frequency of the controller ω_n, based on equation (3.5) and for damping ratio ζ=0.8. This is in general considered to be good for a system, neither too sluggish, nor too stiff. As is discussed in the next section these values for the natural frequency and the damping ratio give also reasonable values for the gains k_1 and k_2.

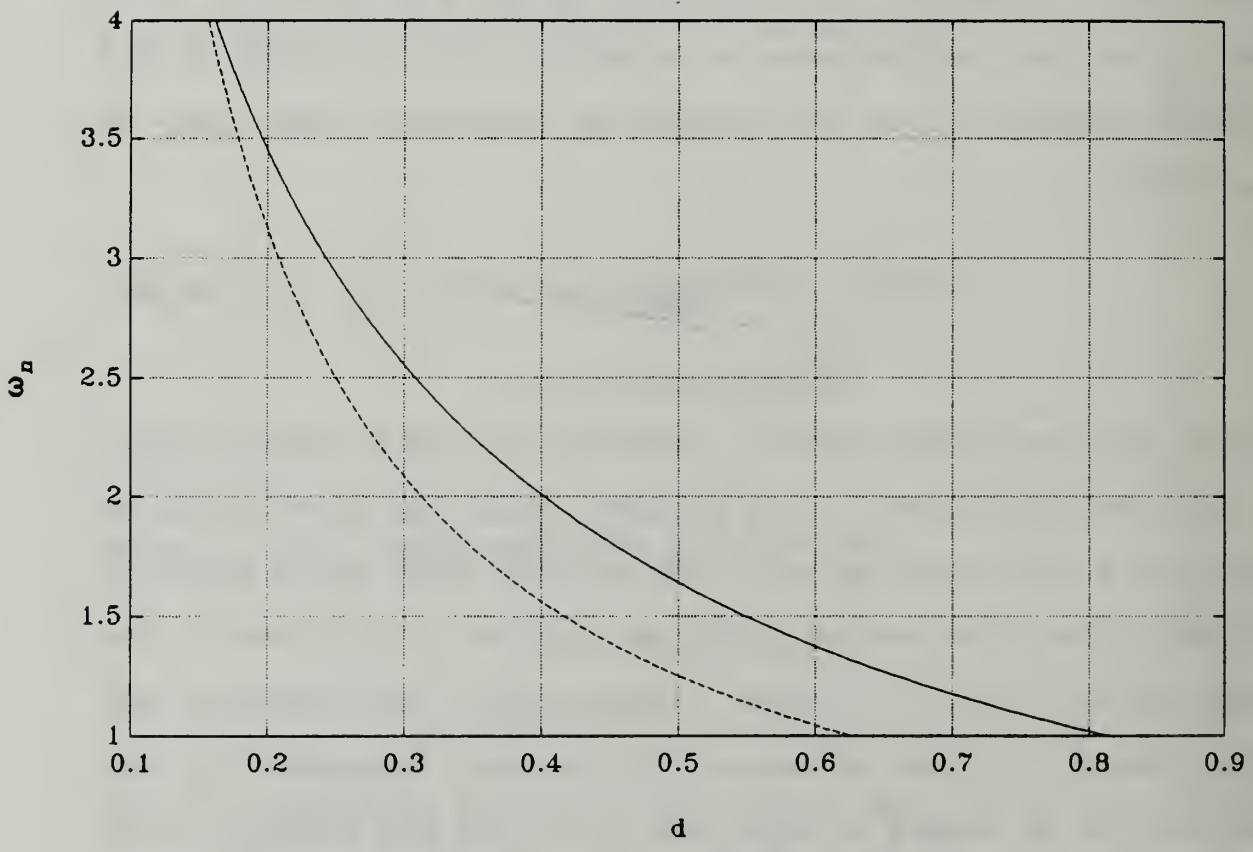

Figure 2. Exact and Approximate Stability Curves for Various ω_n.

B. EXACT CONDITION FOR STABILITY

For an exact calculation we use equations (2.3), (2.8), (2.9) and equation (2.5) which in linearized form appears as:

$$\dot{y}=\psi+v \tag{3.6}$$

This set completes the control law while the guidance scheme is given equation (3.2). Substituting the control law into the equations (2.8), (2.9) and rearranging the terms the system becomes as shown :

$$\dot{\psi}=r \tag{3.7}$$

$$\dot{v}=k_1 b_1 \psi+a_{11} v+(a_{12}+k_2 b_1) r+\frac{k_1 b_1}{d} y \tag{3.8}$$

$$\dot{r}=k_1 b_2 \psi+a_{21} v+(a_{22}+k_2 b_2) r+\frac{k_1 b_2}{d} y \tag{3.9}$$

$$\dot{y}=\psi+v \tag{3.10}$$

Local stability properties are established by the eigenvalues of the stiffness matrix. The characteristic equation of this system is a fourth order polynomial in the form :

$$A\lambda^4+B\lambda^3+C\lambda^2+D\lambda+E=0 \tag{3.11}$$

where :

$$C=C_1+C_2$$

$$D=D_1+D_2$$

with A, B, C_1, C_2, D_1, D_2 and E defined as in the following :

$$A=1.0$$

$$B=-(a_{11}+a_{22})-k_2b_2$$

$$C_1=(a_{11}a_{22}-a_{12}a_{21})+(a_{11}b_2-a_{21}b_1)k_2-k_1b_2$$

$$C_2=-\frac{k_1b_1}{d}$$

$$D_1=(a_{11}b_2-a_{21}b_1)k_1$$

$$D_2=\frac{(a_{22}b_1-a_{12}b_2)k_1}{d}-\frac{k_1b_2}{d}$$

$$E=\frac{(a_{11}b_2-a_{21}b_1)k_1}{d}$$

Routh's criterion for stability determines that loss of stability occurs when the following relation is satisfied :

$$BCD-B^2E-AD^2=0$$

After some algebra a quadratic equation for d occurs as below:

$$\alpha_1d^2+\alpha_2d+\alpha_3=0 \qquad (3.12)$$

where the coefficients α_1, α_2, α_3 are defined as follows :

$$\alpha_1 = BC_1D_1 - D_1^2$$

$$\alpha_2 = BC_1D_2d + BC_2dD_1 - B^2Ed - 2.0D_1D_2d$$

$$\alpha_3 = BC_2dD_2d - D_2^2d^2$$

The positive root of equation (3.12) determines the critical value of the look ahead distance d for stability. For every d > $d_{critical}$ the system is stable and the vehicle will follow the desired path. In the other case when d < $d_{critical}$ the system becomes unstable and the vehicle moves with an oscillatory motion as a result of a complex conjugate pair of eigenvalues with positive real parts that appears in this case. In Figure 2 the continuous line represents the exact stability curve for various values of the ω_n versus the look ahead distance d and for a damping coefficient ζ=0.8. As can be seen the approximate stability curve (dotted line) is very close to the exact and therefore it can be used as a good first approximation in order to provide adequate stability values to the system. In Figure 3 a family of curves was plotted for damping ratios with values of 1.2 to 0.4 with step 0.1 (left to right) for various values of ω_n versus the look ahead distance d. As can be seen the stability area is increasing for higher values of the natural frequency ω_n and for higher values of the damping ratio ζ. When the natural frequency

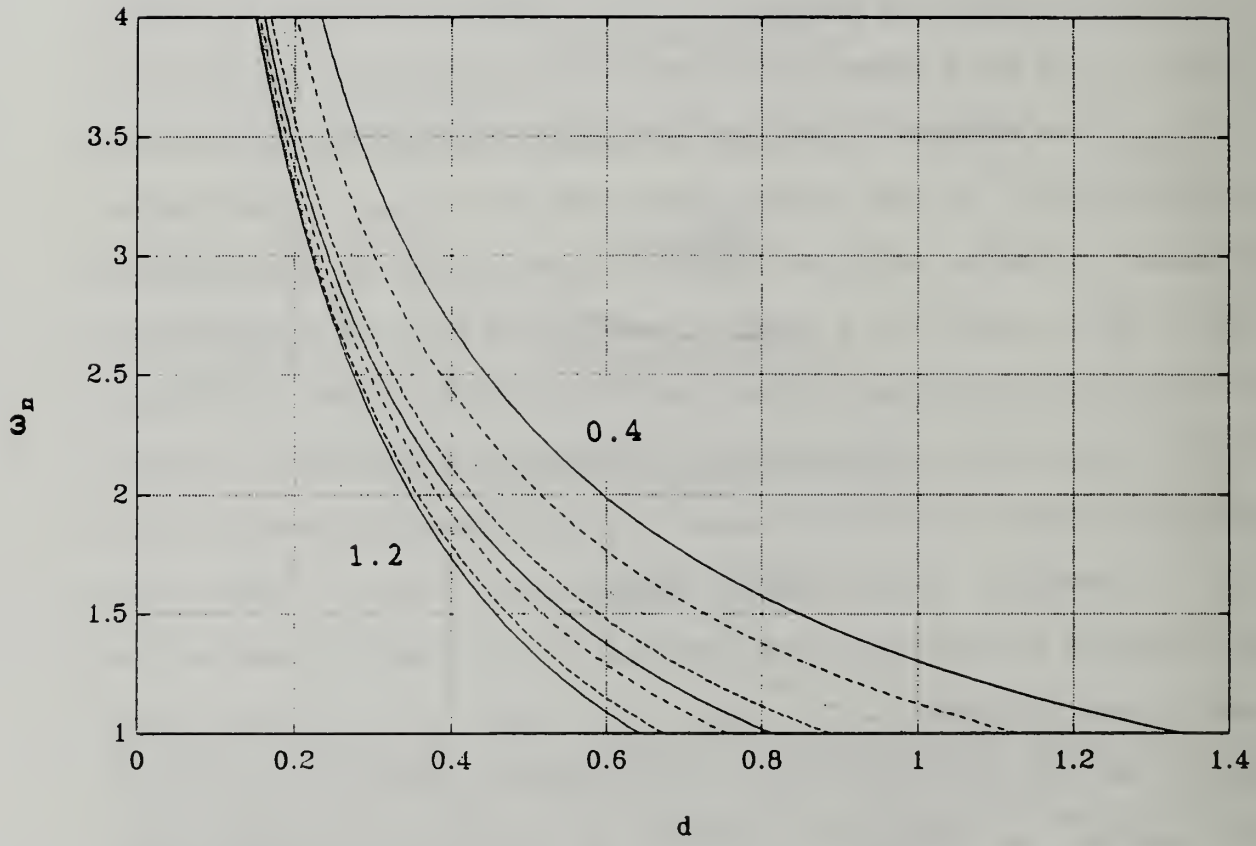

Figure 3. Stability Curves for Various Damping Ratios

though reaches a value of 3.0 or higher, it is observed that the stability curves for the various damping ratios are approaching each other giving almost the same stability area. From Figure 4 it is also observed that for a natural frequency of 3.0 the gains k_1, k_2 are about 3.2 and 1.2 respectively while higher values of ω_n give higher gains without much increase in the stability area. Therefore the above selection of ω_n and ζ to be 3.0 and 0.8 respectively, is considered good. An other way to look at the gains is that for 1 degree of heading error a rudder of 3.2 degrees is required while for 1 degree/sec turning rate a 1.2 rudder angle is required.

C. SENSITIVITY OF STABILITY CURVES

The hydrodynamic coefficients of the vehicle on which all the calculations were based have been determined experimentally. Therefore it is realistic to assume that a degree of error (uncertainty) that enters the calculations and may influence the results. In order to see how much this uncertainty for each coefficient changes the stability curves, a variation of 50% of the nominal value for each hydrodynamic coefficient was assumed and plotted versus the normalized look ahead distance d_n (d_{new}/d_{nom}). The coefficients that found to affect stability were: Y_v, Y_v, N_r and N_r while the rest of the coefficients had no effect. In Figure 5 the variation of the coefficient Y_v was plotted versus the normalized look ahead distance d_n, and as it can be observed a variation of a

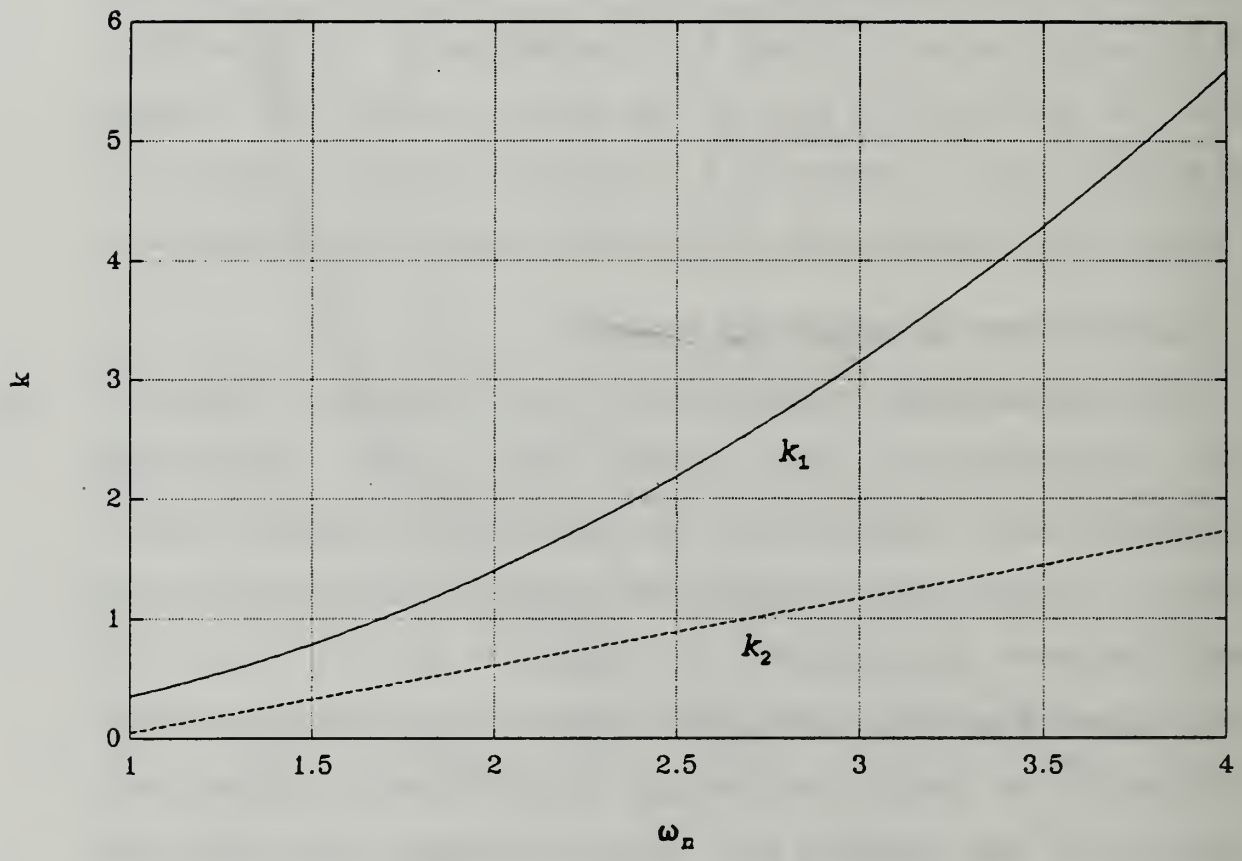

Figure 4. Control Feedback Gains versus ω_n.

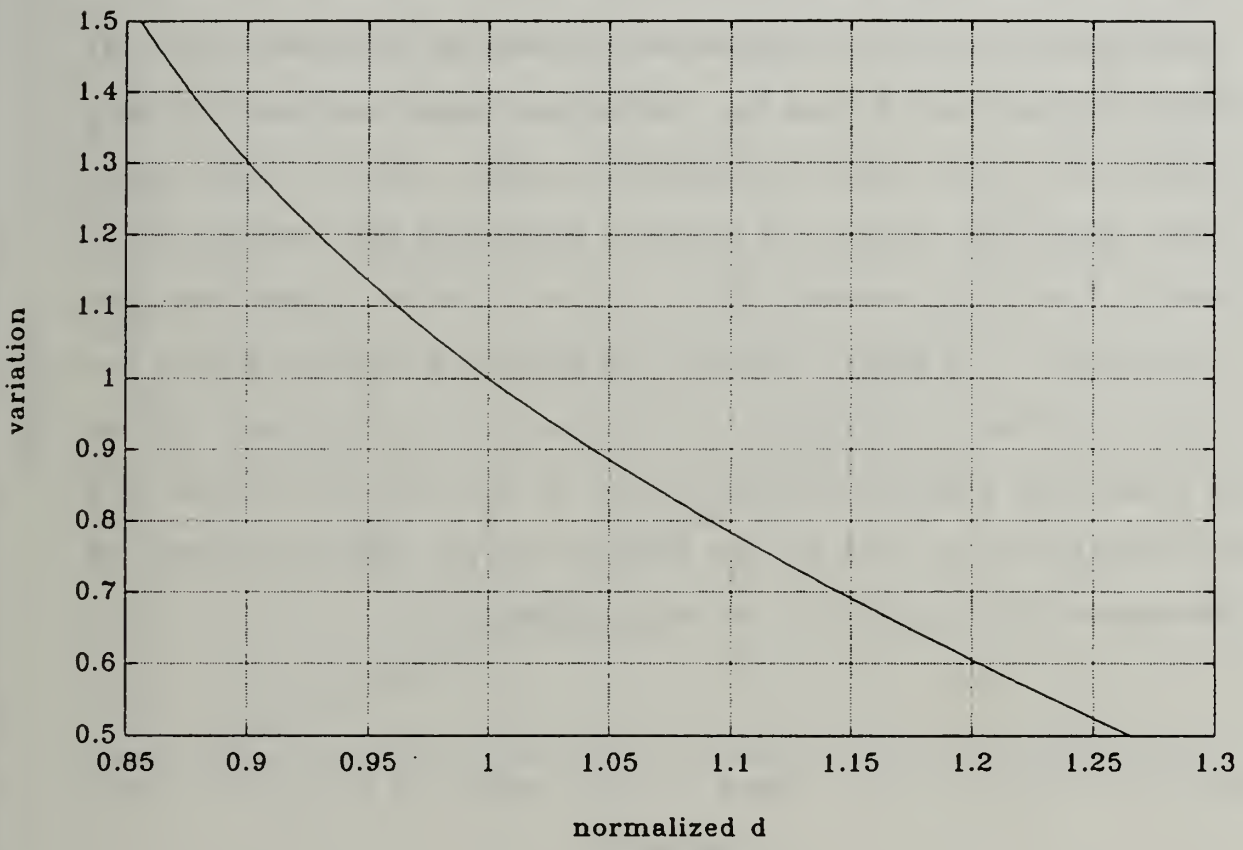

Figure 5. Changes of Critical Look Ahead Distance d for Variation of Y_v

50% of the nominal value gives a total change of about 0.40 for d_n. In Figure 6 the variation of Y_v versus d_n was plotted and for a variation 50% of Y_v a total change of about 0.37 of d_n occurred. In Figure 7 the variation for N_r gives a total change of 0.14 for d_n while in Figure 8 the variation for N_r gives a total change of 0.102 for d_n. As can be seen the variation of Y_v and Y_v gives values for d_n of about four (4) times larger than N_r and N_r, therefore these two coefficients affect the look ahead distance the most. For all the above plots above the values for natural frequency and damping ratio were 3.0 and 0.8 respectively. In an attempt to see how much a variation of ζ and ω_n affects the previous results for Y_v and Y_v the following Figures 9, 10, 11 and 12 are plotted. It can be seen that the maximum deviation in the critical value of d for stability is 40% of its nominal value, while for most of the cases this deviation is well below 20%.

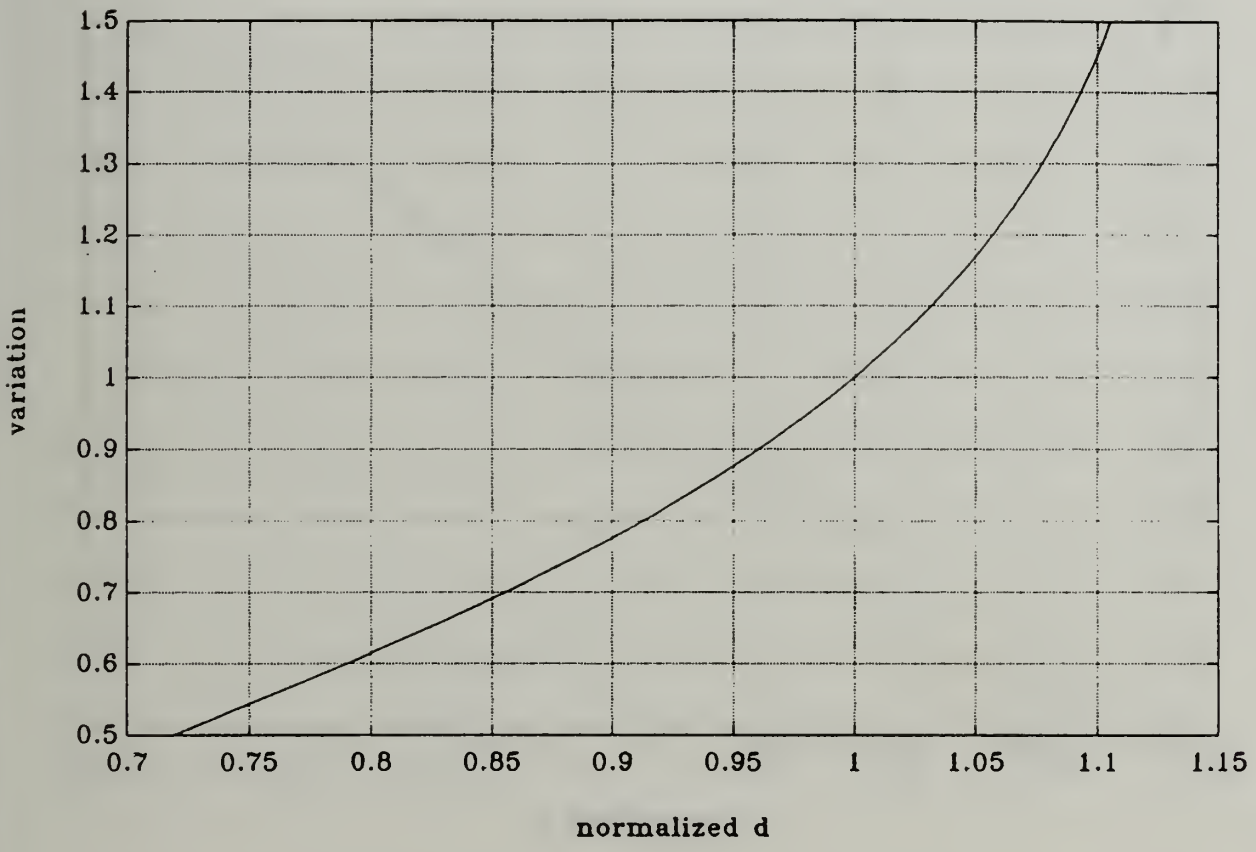

Figure 6. Changes of Critical Look Ahead Distance d for Variation of Y_v.

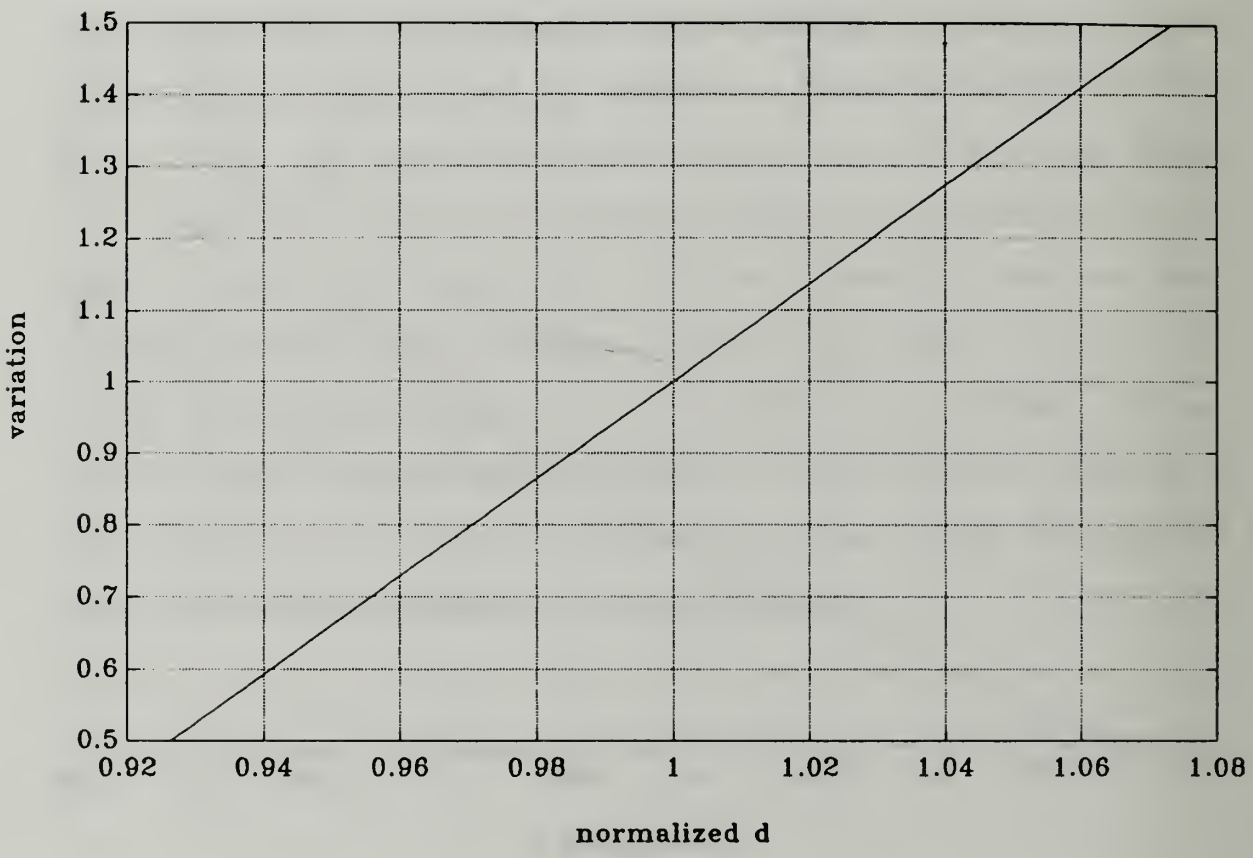

Figure 7. Changes of Critical Look Ahead Distance d for Variation of N_t.

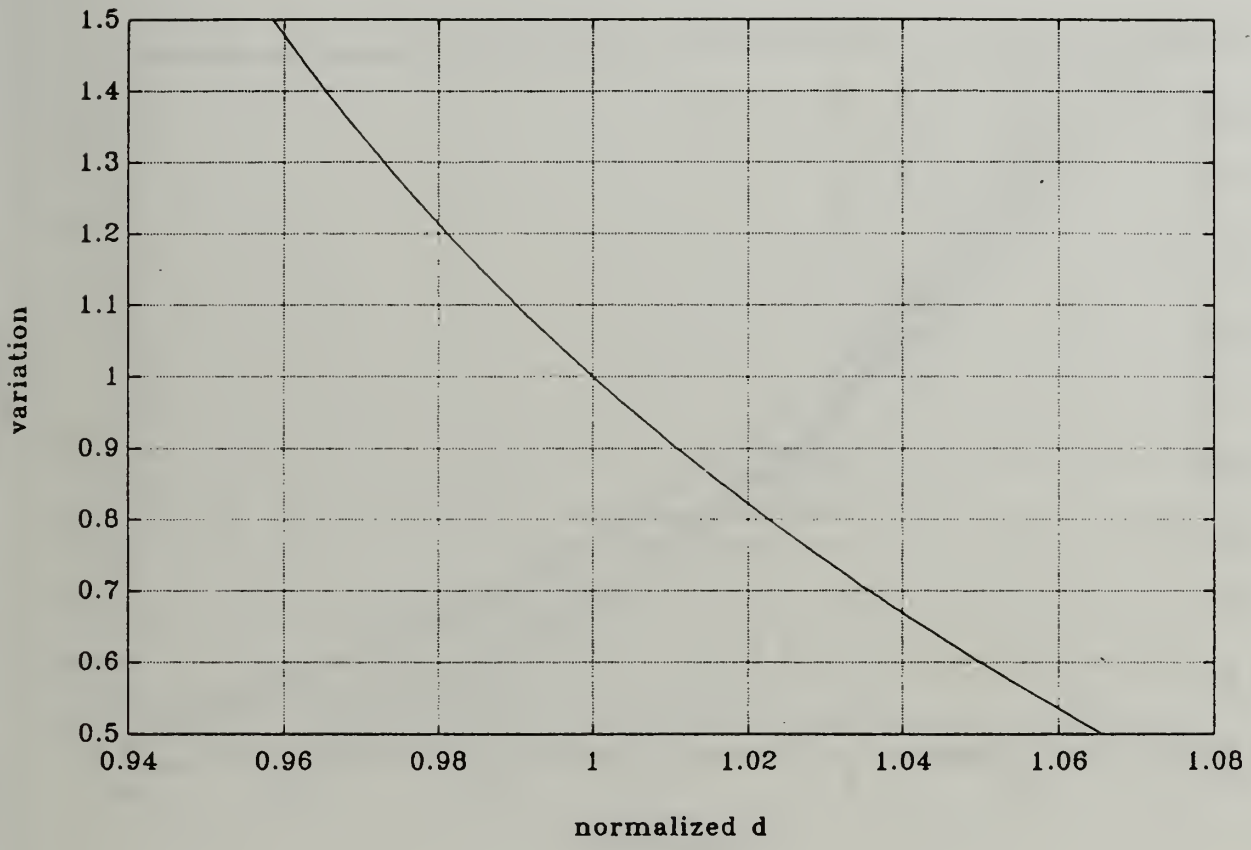

Figure 8. Changes of Critical Look Ahead Distance d for Variation of N_r.

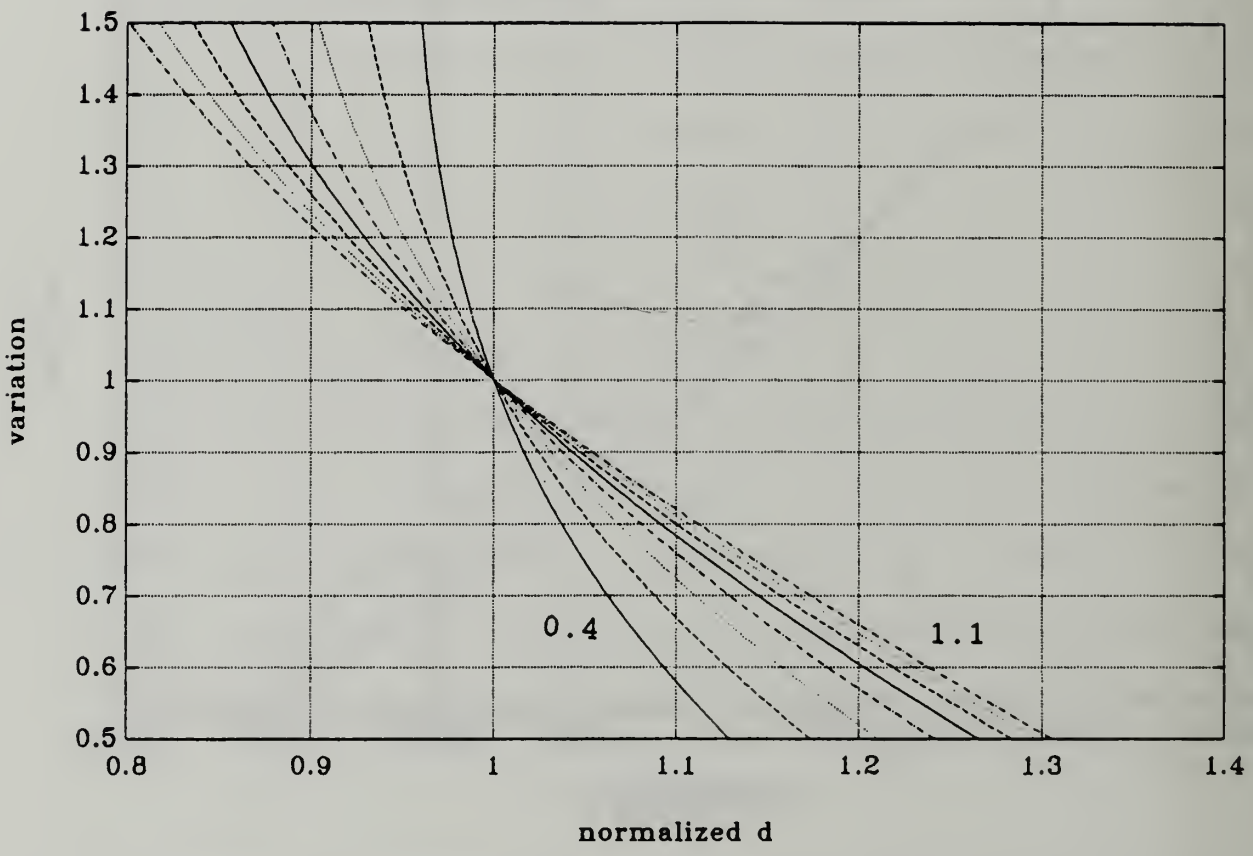

Figure 9. Sensitivity of Critical Look Ahead Distance for Variation in Y_v and for Different Damping Ratios.

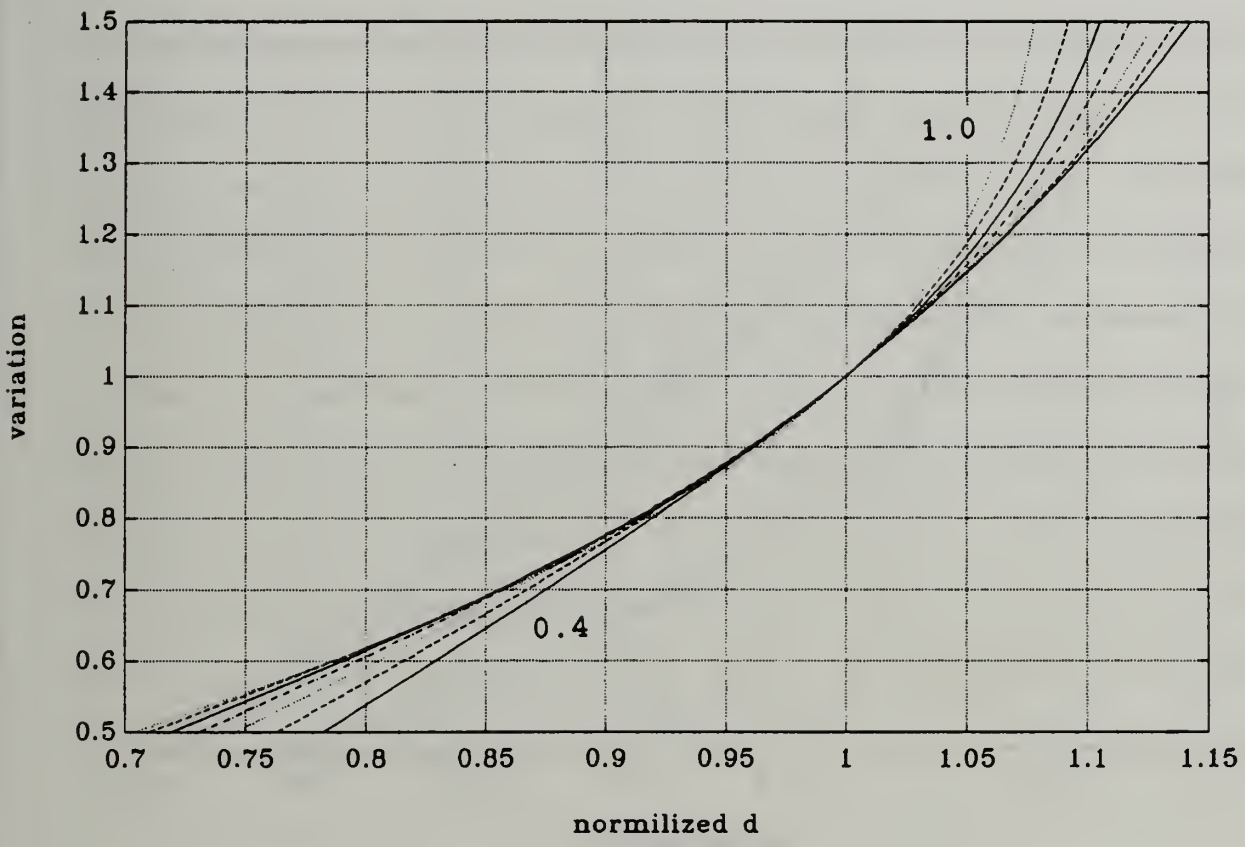

Figure 10. Sensitivity of Critical Look Ahead Distance for Variation in Y_v and for Different Damping Ratios.

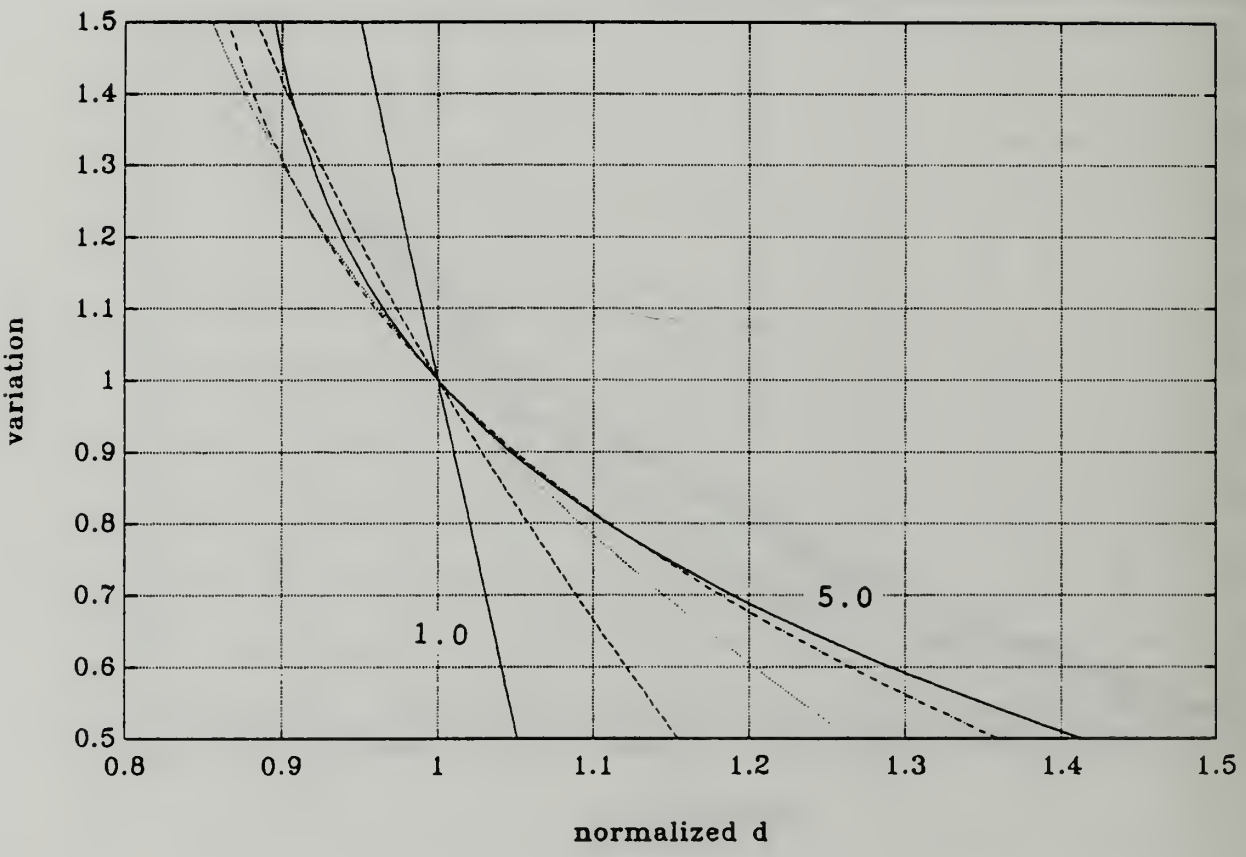

Figure 11. Sensitivity of Critical Look Ahead Distance for Variation in Y_v and for Different Natural Frequencies.

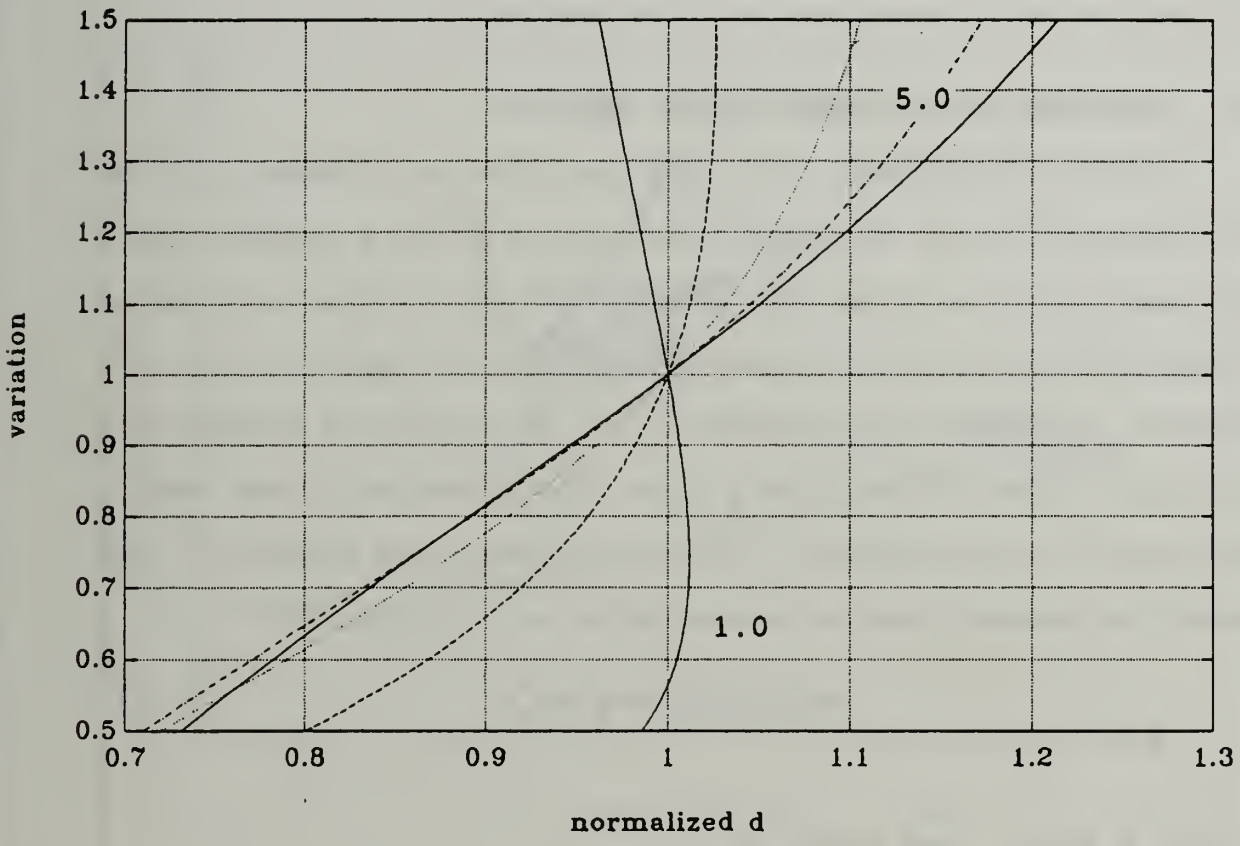

Figure 12. Sensitivity of Critical Look Ahead Distance for Variation in Y_v and for Different Natural Frequencies.

IV. SHIP PATH KEEPING

After satisfying the requirements for stability by selecting proper values for the gains k_1, k_2 and the look ahead distance d as was discussed in the previous section, the ship path keeping problem along straight line segments and circular trajectories is examined in this section.

A. STRAIGHT LINE SEGMENTS PATH KEEPING

So far the control law with the guidance scheme is given by equation (2.18) which in steady state gives a heading angle of zero (0). In order for the vehicle to follow a straight line path between two given points (x_1, y_1) and (x_2, y_2) the control law must be modified. If α, is the angle between the straight line (P) and the x axis; the distance y that enters the control law becomes y´, as can be seen from Figure 13. The modified control law is shown below :

$$\delta = k_1 \sin(\psi - \sigma - \alpha) + k_2 r \qquad (4.1)$$

where σ and y´ are given by :

$$\sigma = \psi_{com} = -\arctan\left(\frac{\acute{y}}{d}\right)$$

$$\acute{y} = (y - y_1)\cos\alpha - (x - x_1)\sin\alpha$$

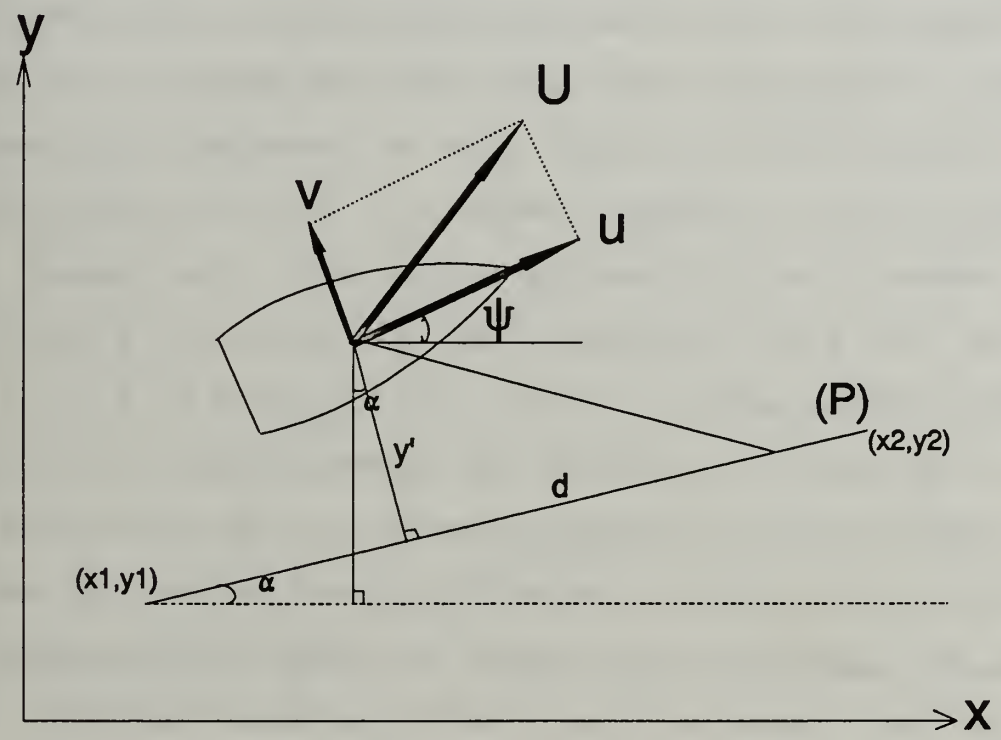

Figure 13. Guidance Scheme for Straight Line Path

At steady state now the heading angle becomes α and the vehicle follows the straight line (P). The simulation program takes into account all the possible different orientations of (P) by modifying the angle α properly for each case. In Figure 14 we can see the simulated path of the vehicle (continuous line) and the reference path (dotted) for a two way point path. The reference line is a quarter of a circle while the two points have coordinates (0,0) and (10,10) respectively. In Figure 15 the rudder time history for the above two way point path keeping for a straight line is presented. The maximum value is about -23 degrees, which is a saturation point since the rudder is not allowed to exceed this value (exact 0.4 radian or 22.923 degrees). In the following Figures 16 through 23 the number of points was increased to 3, 5, 7 and 13, in an effort to approach the reference path, by dividing the reference path in equally spaced arcs and following the straight line segments defined by these points. As can be seen, by increasing the number of points the vehicle can follow the reference path with increased accuracy. The drawbacks of simply increasing the number of discretization points are two. First, we are placing increased demands on storage and memory for the on-board processor. Second, even when satisfactory path accuracy is achieved by using a large number of way points the rudder activity is very excessive and erratic. Therefore if the vehicle must follow a circular arc

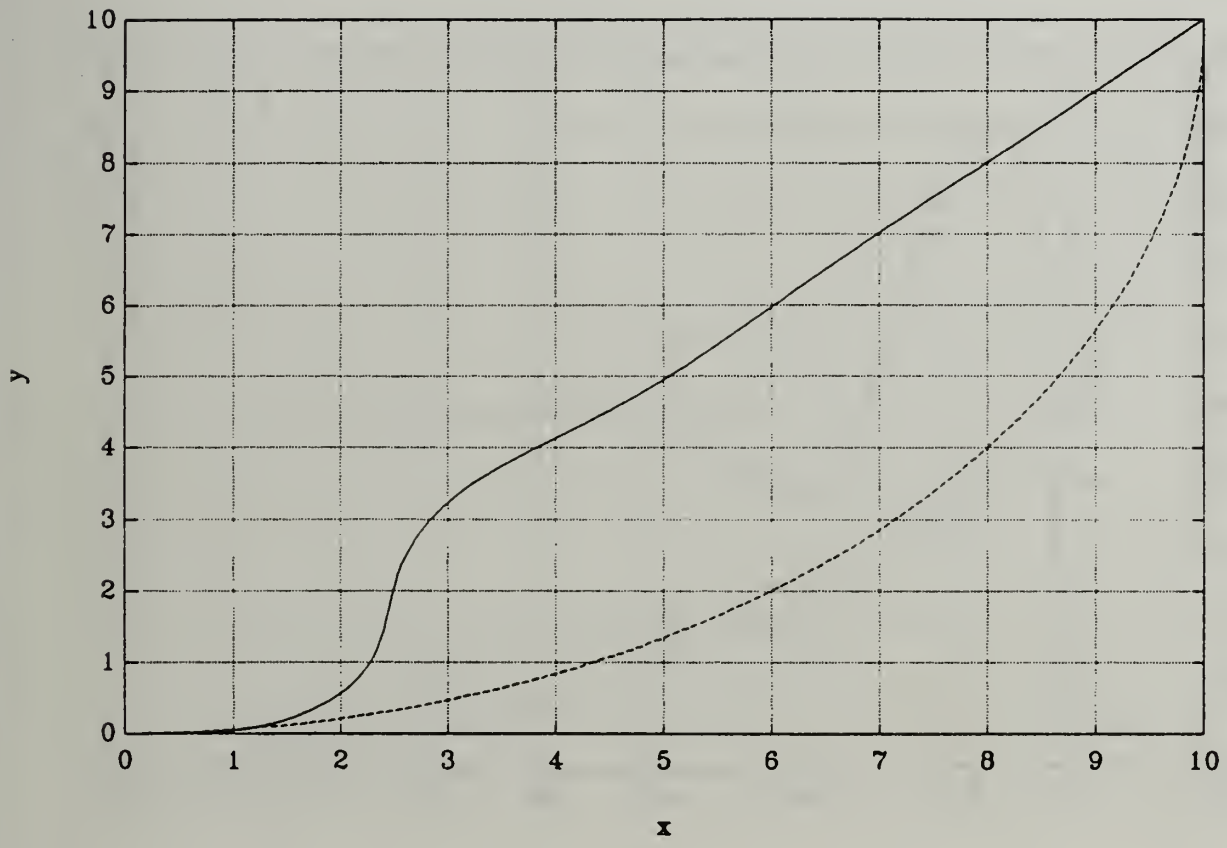

Figure 14. Simulation for Reference and Vehicle Path for 2 Way Points.

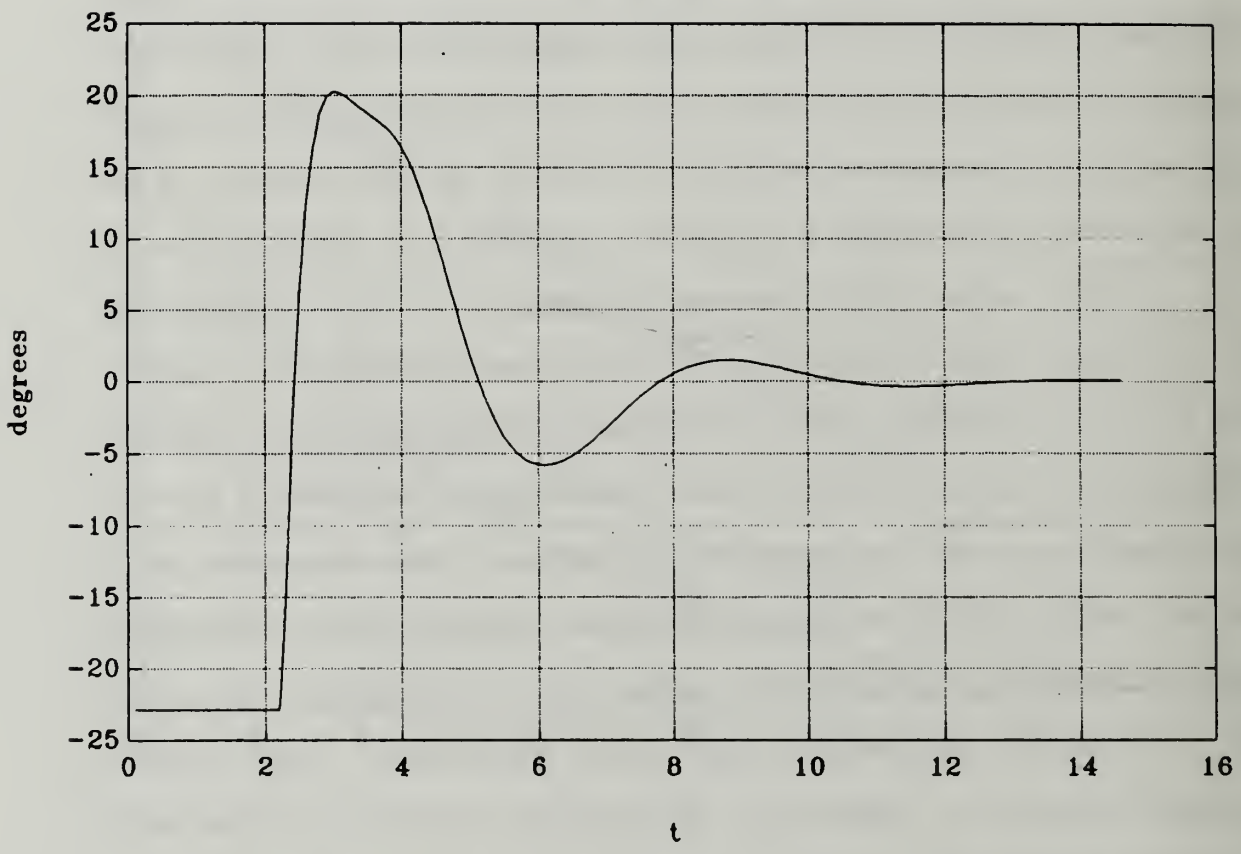

Figure 15. Rudder Time History for 2 Way Point Path.

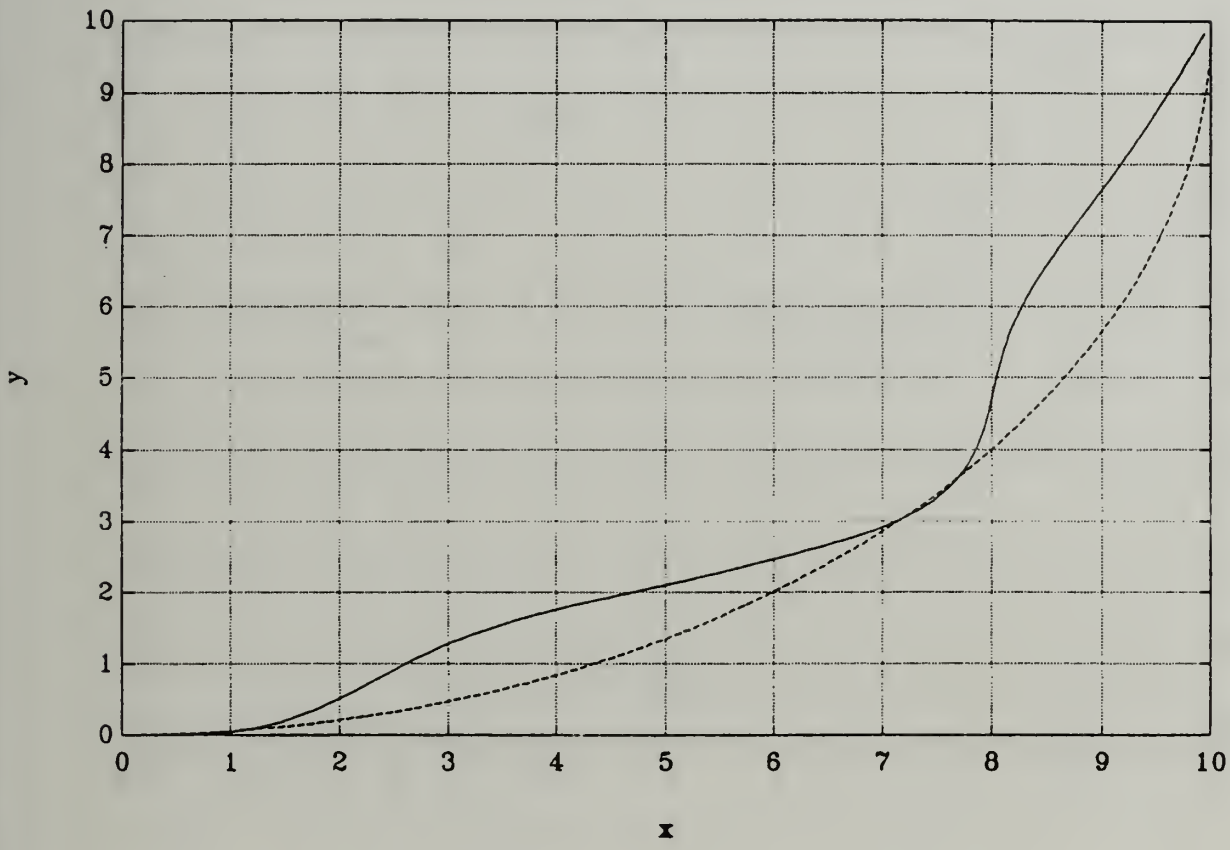

Figure 16. Simulation for Reference and Vehicle Path for 3 Way Points.

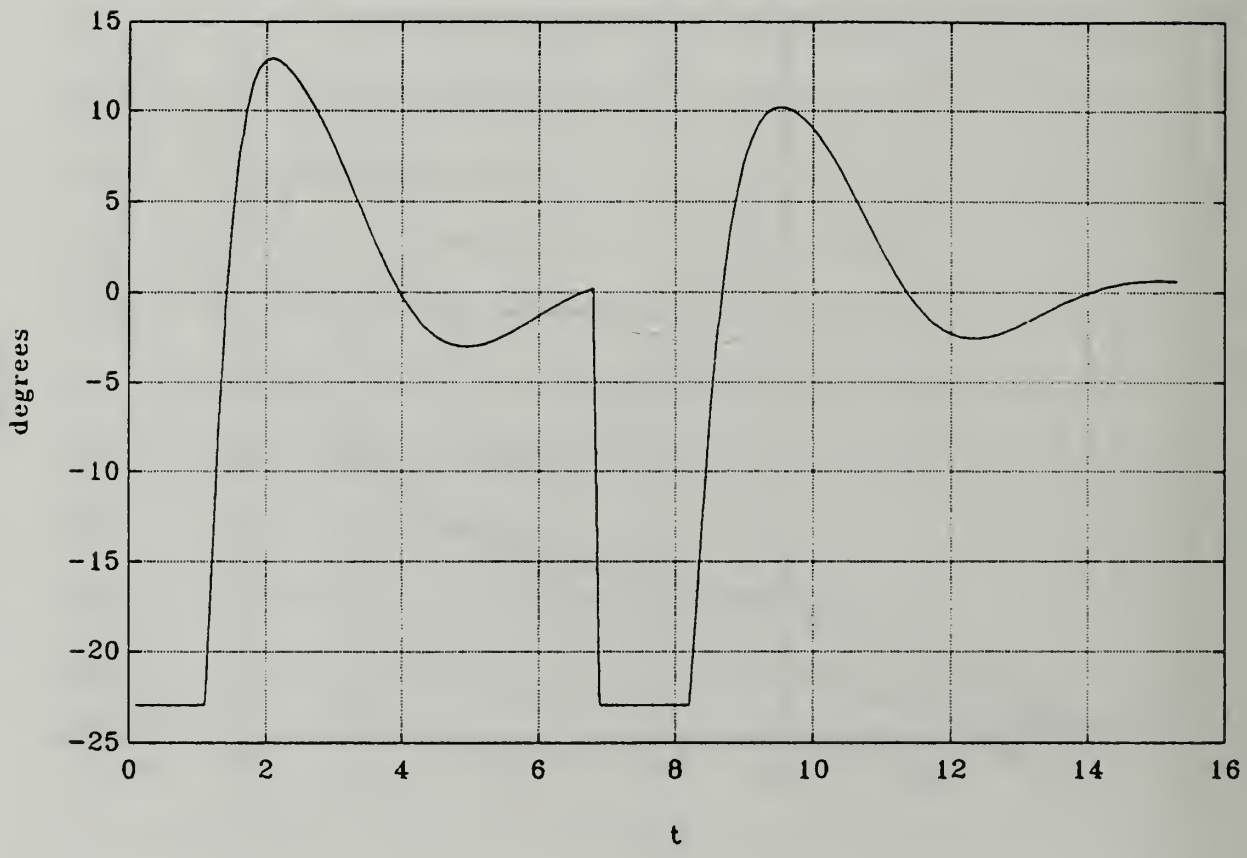

Figure 17. Rudder Time History for 3 Way Point Path.

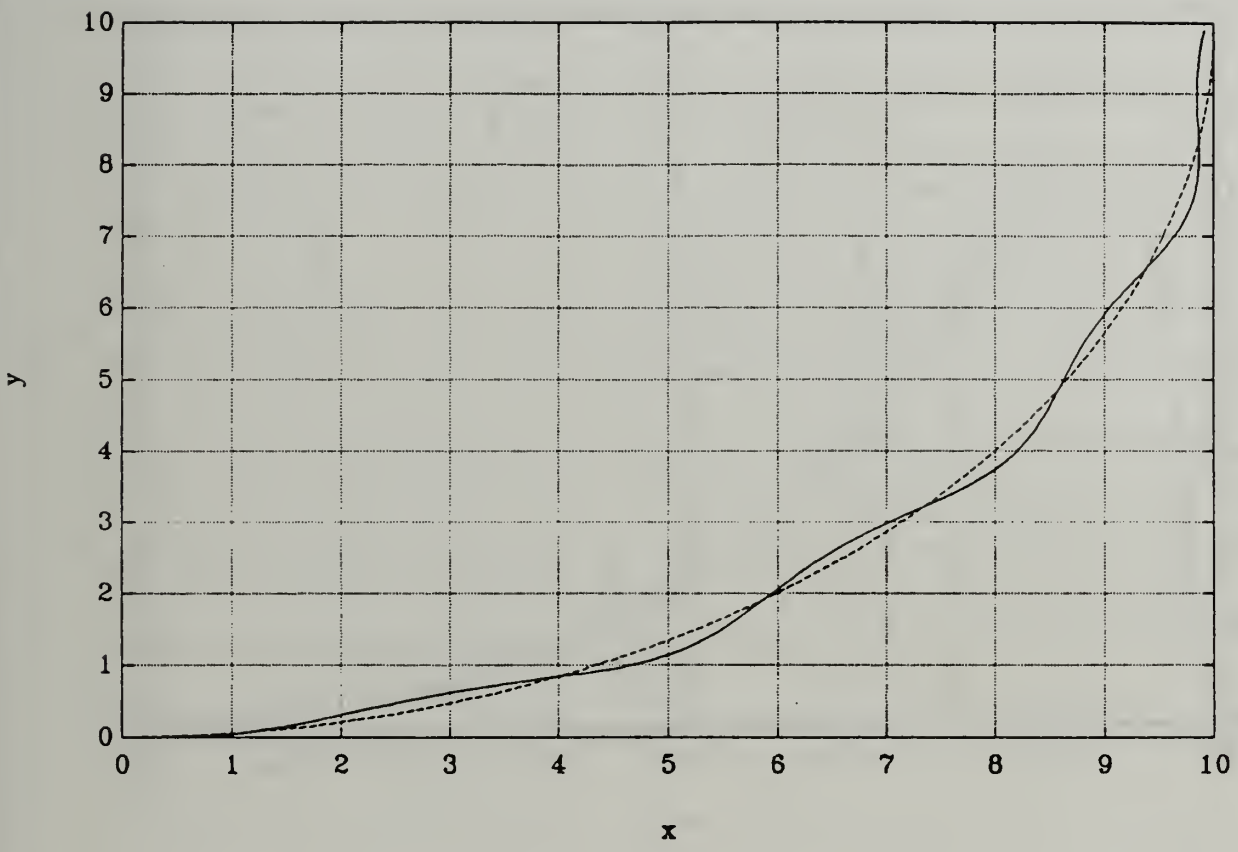

Figure 18. Simulation for Reference and Vehicle Path for 5 Way Points.

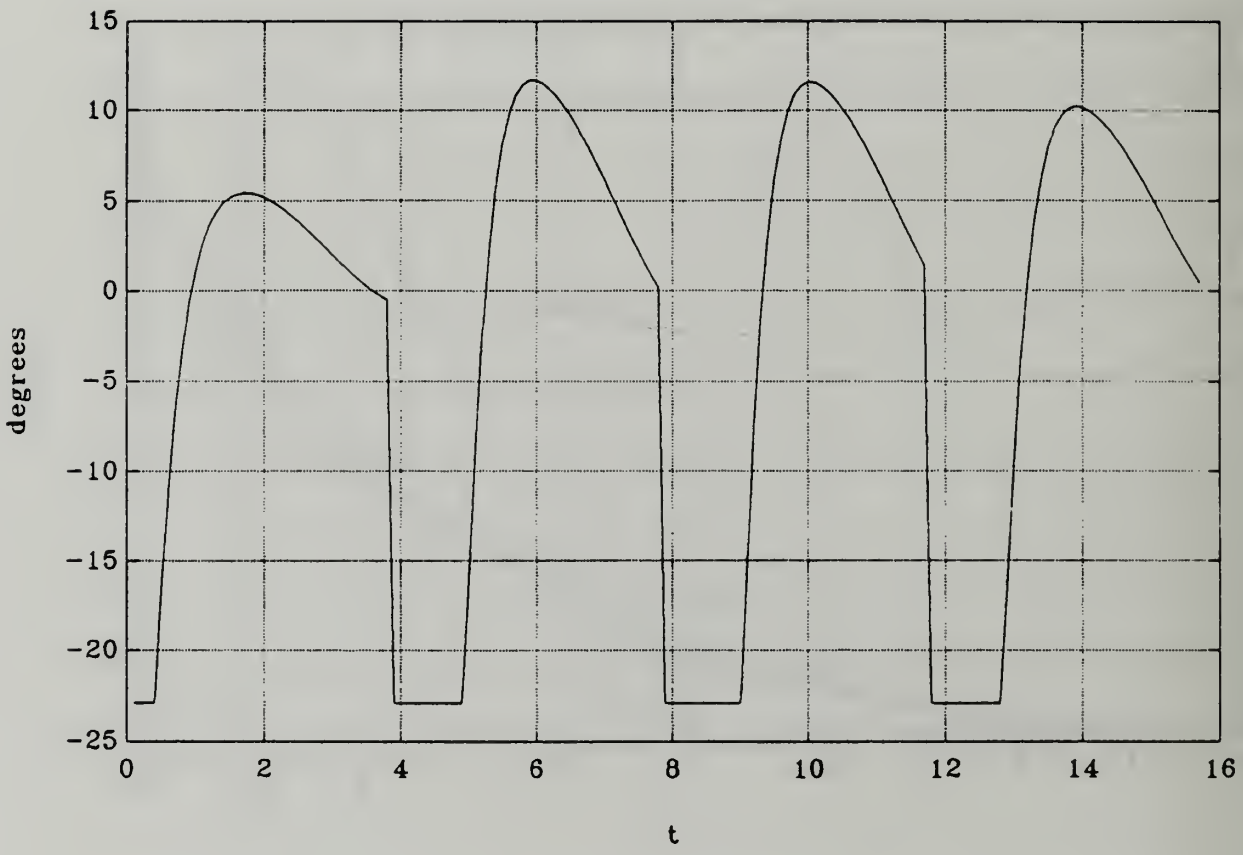

Figure 19. Rudder Time History for 5 Way Point Path.

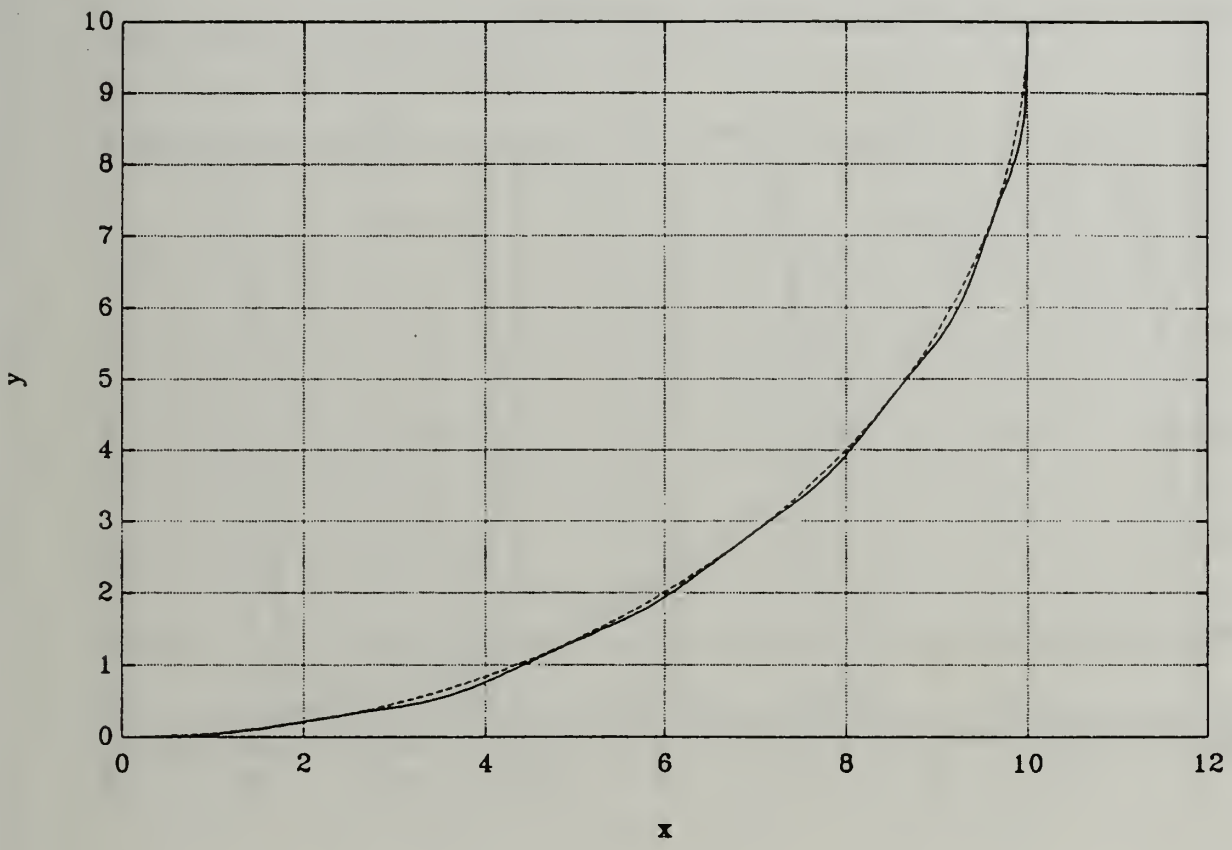

Figure 20. Simulation for Reference and Vehicle Path for 7 Way Points.

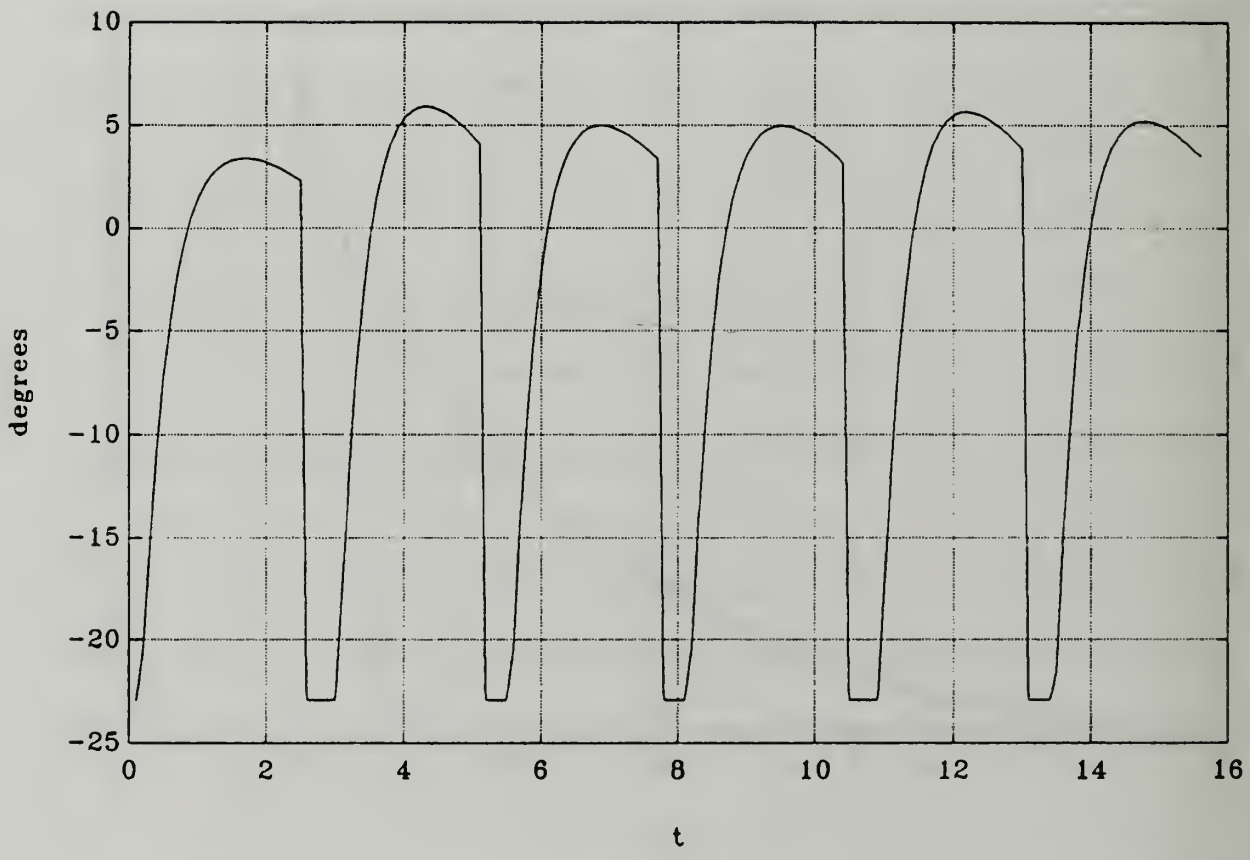

Figure 21. Rudder Time History for 7 Way Point Path.

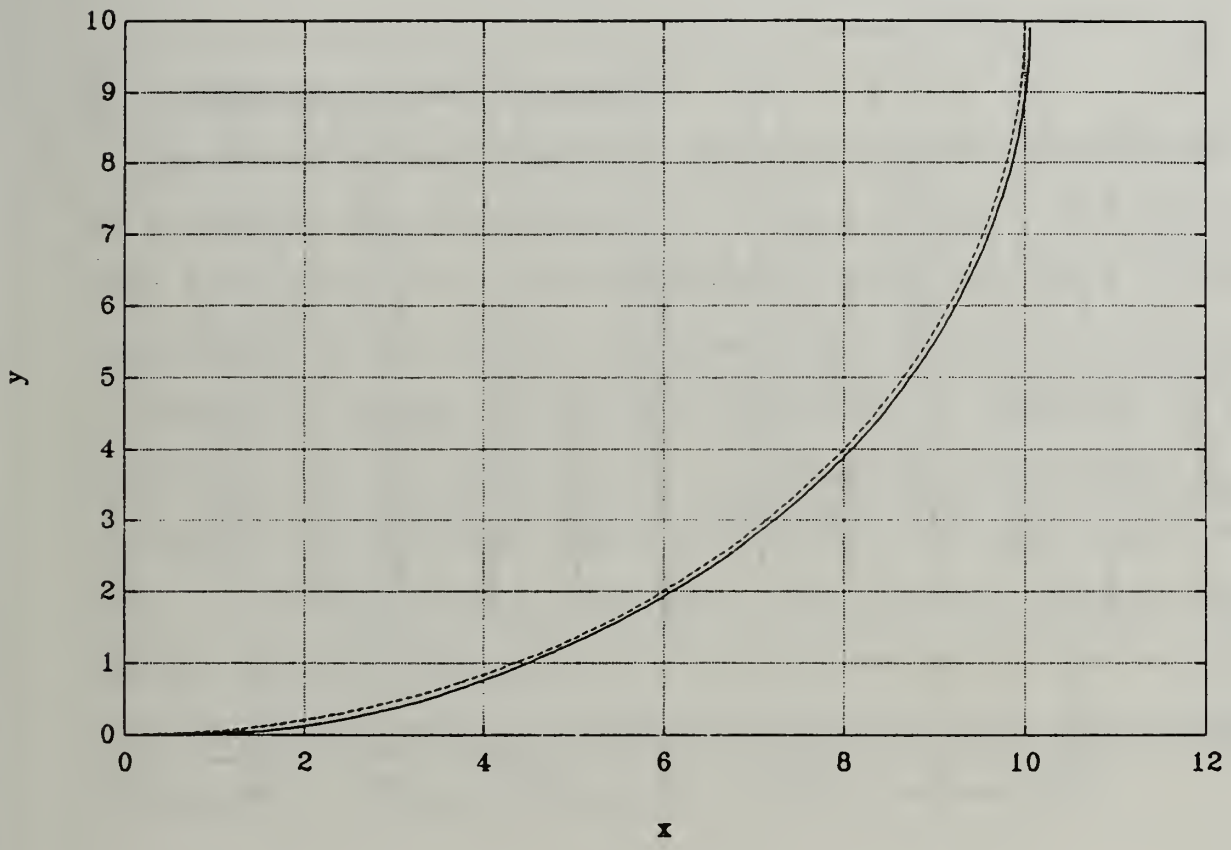

Figure 22. Simulation for Reference and Vehicle Path for 13 Way Points.

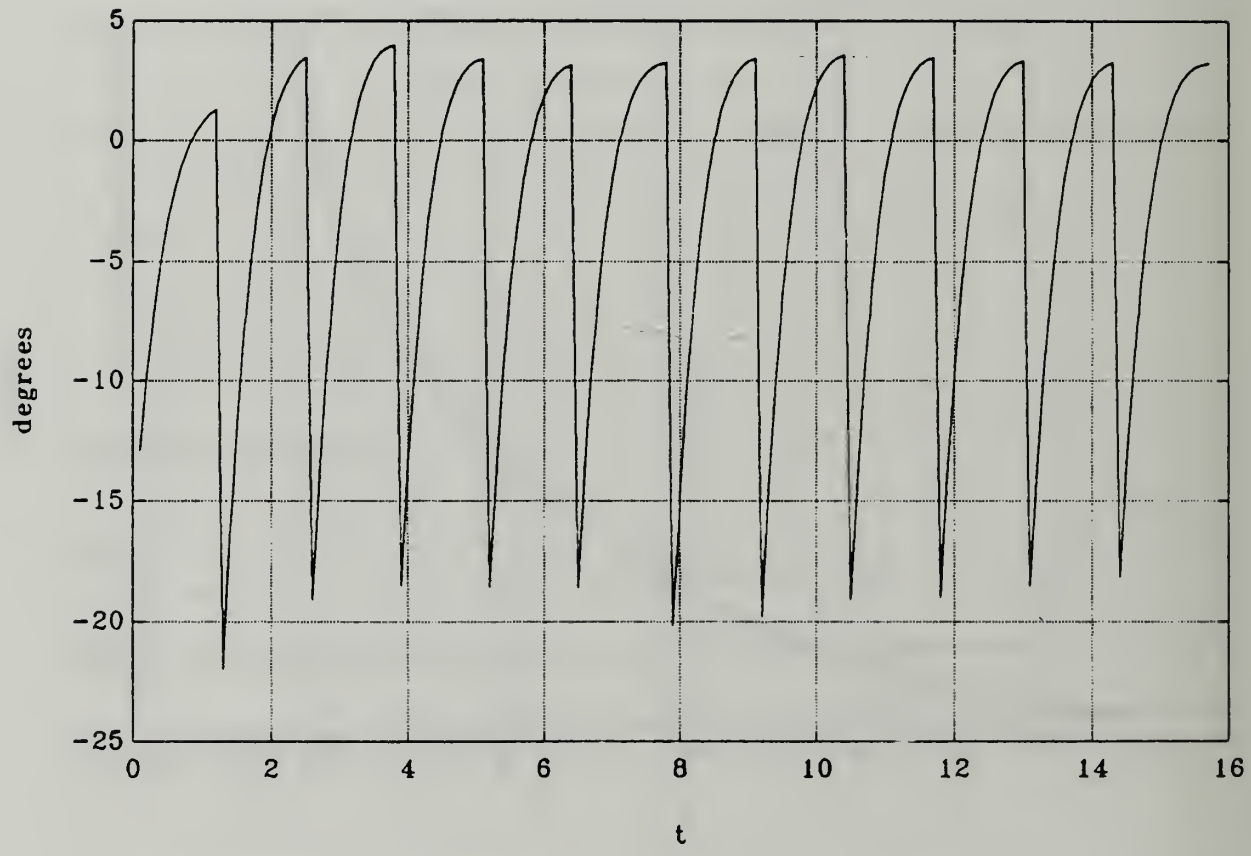

Figure 23. Rudder Time History for 13 Way Point Path.

path, doing so by following small straight line segments is not the best way since a large number of points is needed and the rudder is doing a considerable amount of effort. This means that a different way must be developed such that the vehicle at any time t is following the circular arc instead of a straight line.

B. CIRCULAR ARC PATH KEEPING

By demanding the vehicle to follow a certain circular path in a certain way, clockwise or counterclockwise, the angle σ that enters the control law is going to vary depending on the orientation of the vehicle relative to the center of the path. It needs to change values from positive to negative in a consistent way with the control law requirements, in order for the vehicle to negotiate the assigned path. For this reason we need, to properly modify the control law for each case of the vehicle motion (cw, ccw) and for each quarter of the circle separately. Defining by A the distance from point V to point O, Figure 24, A is given by the following expression :

$$A=[(x-x_0)^2+(y-y_0)^2]^{\frac{1}{2}} \qquad (4.2)$$

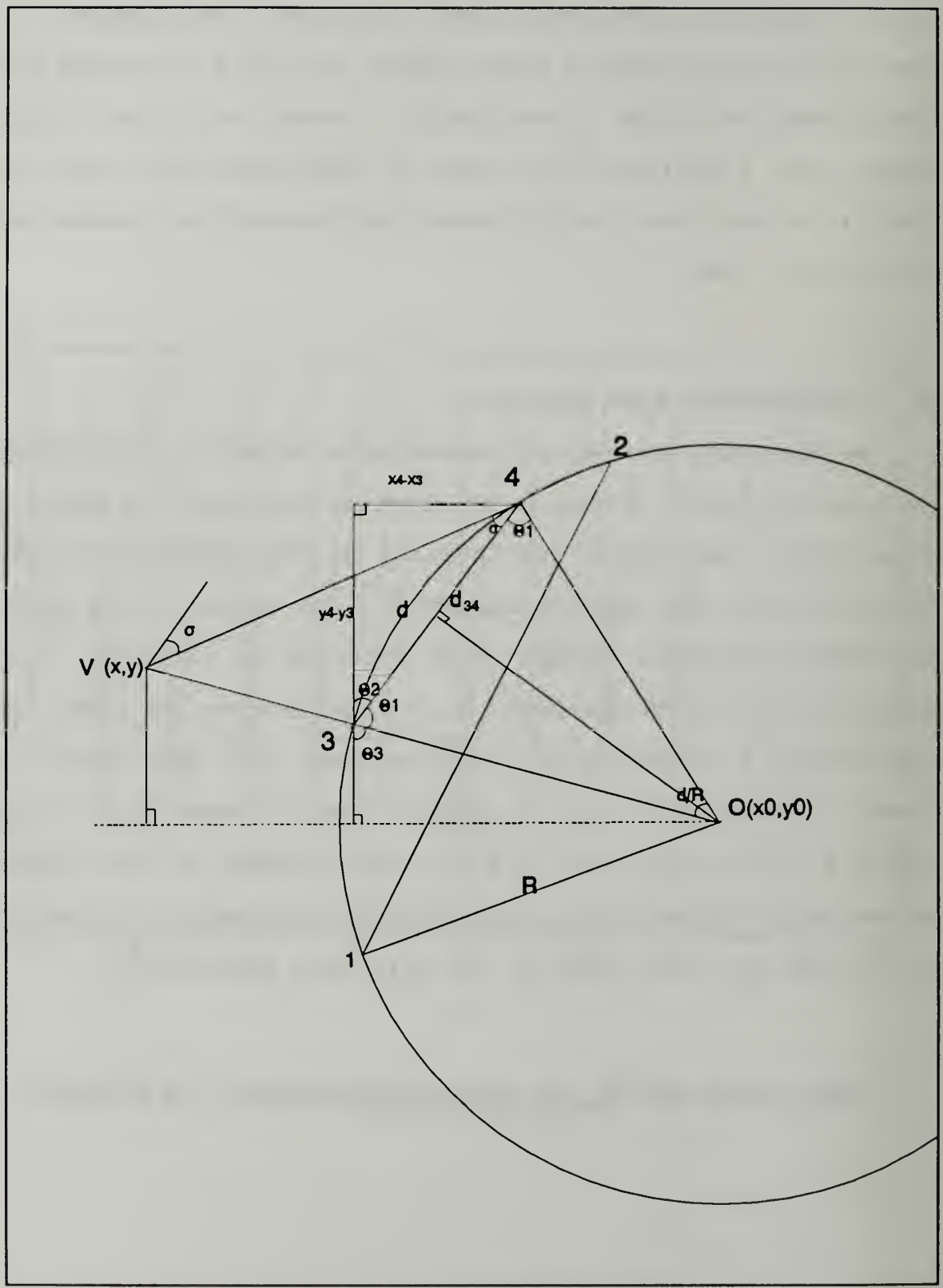

Figure 24. Guidance Scheme for CW Motion.

The radius of the circular path R can also be expressed as :

$$R=[(x_1-x_0)^2+(y_1-y_0)^2]^{\frac{1}{2}} \tag{4.3}$$

From similar triangles the following relations yield the coordinate values of point 3 :

$$x_3=x_0+\frac{R}{A}(x-x_0) \tag{4.4}$$

$$y_3=y_0+\frac{R}{A}(y-y_0) \tag{4.5}$$

Equations (4.2), (4.3), (4.4) and (4.5) are valid for CW or CCW motion and for any possible orientation of the vehicle. The equations that are valid for each separate case are presented below.

1. CLOCKWISE MOTION

a) For the fourth quarter of the circle and from Figure 24 the angle θ_3 can be written as :

$$\theta_3=\arcsin\left(\frac{x_0-x_3}{R}\right) \tag{4.6}$$

The angle θ_2 also appears as follows :

$$\theta_2 = \pi - \theta_1 - \theta_3 \qquad (4.7)$$

where θ_1 can be written as :

$$\theta_1 = \frac{\pi}{2} - \frac{1}{2}\left(\frac{d}{R}\right) \qquad (4.8)$$

For this case d is the length of the arc from point 3 to point 4, while d_{34} is the distance between points 3 and 4 and can be written as follows :

$$d_{34} = 2R\sin\left(\frac{d}{2R}\right) \qquad (4.9)$$

The coordinates of point 4 now since d_{34}, x_3 and y_3 are known can be evaluated from trigonometry as follows :

$$x_4 = x_3 + d_{34}\sin\theta_2 \qquad (4.10)$$

$$y_4 = y_3 + d_{34}\cos\theta_2 \qquad (4.11)$$

The distance B from point V to point 4 is :

$$B=[(x-x_4)^2+(y-y_4)^2]^{\frac{1}{2}} \tag{4.12}$$

Defining the angle γ as following :

$$\gamma=\sigma+\theta_1 \tag{4.13}$$

and performing the law of sines in the triangle (OV4), γ is given by :

$$\gamma=\arcsin[\sin(\frac{d}{R})\frac{A}{B}] \tag{4.14}$$

Substituting angle γ, equation (4.13) gives angle σ. For the control law the angle α (the angle that d_{34} makes with the horizontal) is also needed and this is given by :

$$\alpha=\frac{\pi}{2}-\theta_2 \tag{4.15}$$

Finally the control law takes the following form :

$$\delta=k_1\sin(\psi-\alpha+\sigma)+k_2 r \tag{4.16}$$

Taking the sine of the total heading angle accounts for the approximations of linearization that were made in selecting

the gains k_1, k_2. The above form of the control law is valid for all of the quarters of the clockwise motion, while for the counterclockwise motion, angle σ is reversing sign as we will see later. The equations (4.7), (4.8), (4.9), (4.12), (4.13) and (4.14) are also valid for all the cases.

b) For the third quarter, the angle θ_3 is given by the following equation :

$$\theta_3=\arcsin\left(\frac{y_0-y_3}{R}\right)$$

The coordinates of point 4 are now given by the following expressions :

$$x_4=x_3-d_{34}\cos\theta_2$$

$$y_4=y_3+d_{34}\sin\theta_2$$

Finally, the angle α that enters the control law appears as below :

$$\alpha=\pi-\theta_2$$

c) For the second quarter, θ_3 is given by the following equation :

$$\theta_3 = \arcsin\left(\frac{x_3 - x_0}{R}\right)$$

while x_4, y_4 are given as :

$$x_4 = x_3 - d_{34}\sin\theta_2$$

$$y_4 = y_3 - d_{34}\cos\theta_2$$

For this case, the angle α appear as :

$$\alpha = \frac{3}{2}\pi - \theta_2$$

d) Similarly, for the case of the first quarter :

$$\theta_3 = \arcsin\left(\frac{y_3 - y_0}{R}\right)$$

$$x_4 = x_3 + d_{34}\cos\theta_2$$

$$y_4 = y_3 - d_{34}\sin\theta_2$$

$$\alpha = -\theta_2$$

2. COUNTERCLOCKWISE MOTION

From Figure 25 it can be seen that the equations (4.2), (4.3), (4.4), (4.5), (4.7), (4.8), (4.9), (4.12), (4.13), (4.14) continues to hold.

a) For the fourth quarter, Figure 25, angle θ_3, x_4, y_4 and α appear as follows :

$$\theta_3 = \arcsin\left(\frac{y_3 - y_0}{R}\right)$$

$$x_4 = x_3 - d_{34}\cos\theta 2$$

$$y_4 = y_3 - d_{34}\sin\theta_2$$

$$\alpha = \pi + \theta_2$$

The control law for this and also for the other three cases of the counterclockwise motion appears as below :

$$\delta = k_1 \sin(\psi - \alpha - \sigma) + k_2 r$$

b) For the third quarter the following relations hold :

$$\theta_3 = \arcsin\left(\frac{x_0 - x_3}{R}\right)$$

$$x_4 = x_3 + d_{34}\sin\theta_2$$

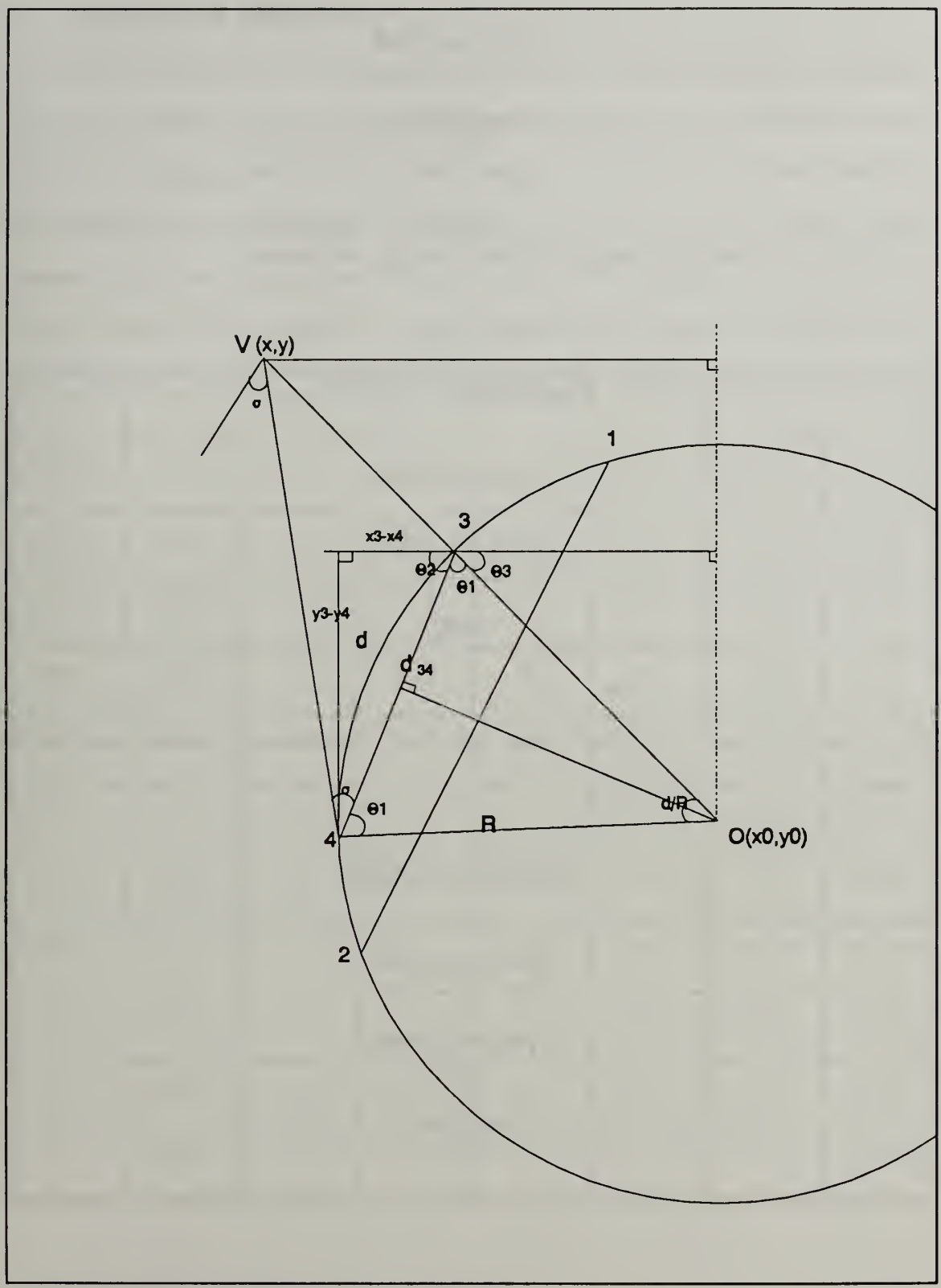

Figure 25. Guidance Scheme for CCW Motion.

$$y_4 = y_3 - d_{34}\cos\theta_2$$

$$\alpha = \frac{3}{2}\pi + \theta_2$$

c) For the second quarter similarly we have :

$$\theta_3 = \arcsin\left(\frac{y_0 - y_3}{R}\right)$$

$$x_4 = x_3 + d_{34}\cos\theta_2$$

$$y_4 = y_3 + d_{34}\sin\theta_2$$

$$\alpha = \theta_2$$

d) Finally for the first quarter the following equations hold:

$$\theta_3 = \arcsin\left(\frac{x_3 - x_0}{R}\right)$$

$$x_4 = x_3 - d_{34}\sin\theta_2$$

$$y_4 = y_3 - d_{34}\cos\theta_2$$

$$\alpha = \frac{\pi}{2} + \theta_2$$

C. SIMULATION RESULTS

By introducing the modifications of the previous chapter into the simulation program, the necessity of providing the simulation program with a set data, in order for the vehicle to perform the desired commands, occurred. The table below summarizes the required data, for the simulation program, for a test case of a figure - eight manoeuvre shown in Figure 28.

x	●	TURN	○	PATH	ICCW	x_0	y_0
10.0	0.0	0	1.0	0	0	0.0	0.0
15.0	10.0	0	0.1	0	0	0.0	0.0
10.0	15.0	0	0.1	1	1	15.0	10.0
8.0	15.0	0	0.1	0	0	0.0	0.0
3.0	20.0	0	0.1	1	0	8.0	20.0
8.0	25.0	0	0.1	1	0	8.0	20.0
13.0	20.0	0	0.1	1	0	8.0	20.0
8.0	15.0	0	0.1	●	0	8.0	20.0
8.0	15.0	0	0.1	0	0	0.0	0.0
1.0	10.0	0	0.1	1	1	6.0	10.0
6.0	5.0	0	0.1	1	1	6.0	10.0

In the first two columns the coordinates x, y of the next way point are given. In the third column, the variable TURN selects how the vehicle is going to change its direction for the next way point in order to have a smooth change in course with less overshoot. There are two ways, 1 for circle, 0 for distance. When 1 is selected the vehicle must enter a circle with a preselected target radius centered at the way point before it starts heading for the next one. When 0 is selected the vehicle turns when it reaches a preselected target distance from the heading point. The fourth column, D, determines this preselected radius or distance that the vehicle has to approach the way point before it starts turning for the next one. The fifth column determines if the desired path is a circular arc (1) or a straight line segment (0). In the sixth column, the (ICCW) index determines if the desired path is in the counterclockwise direction CCW for 1, or clockwise CW for 0. This is the case, of course, when the fifth column is 1 (circular path). Finally, the two last columns give the coordinates of the center of the circular arc if this is the case. In an effort to make a comparison with the previous case of Figures 22 and 23, the vehicle path was plotted for a two way circular path. From Figure 26 can be seen that the reference path was achieved with the use of only two points instead of 13. From Figure 27 we can also see that the rudder steers the vehicle in a smooth way achieving much smaller values than any other case of straight line segments.

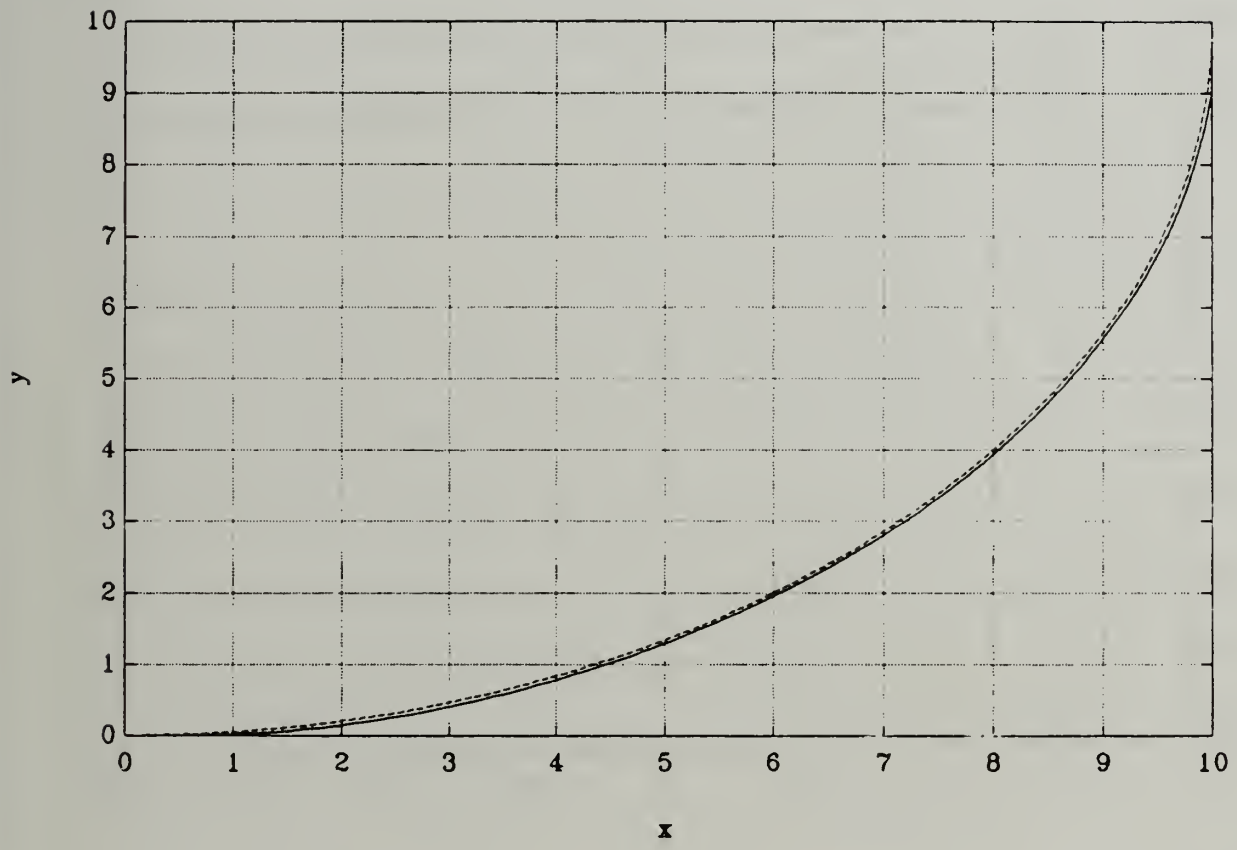

Figure 26. Actual and Reference Vehicle Path.

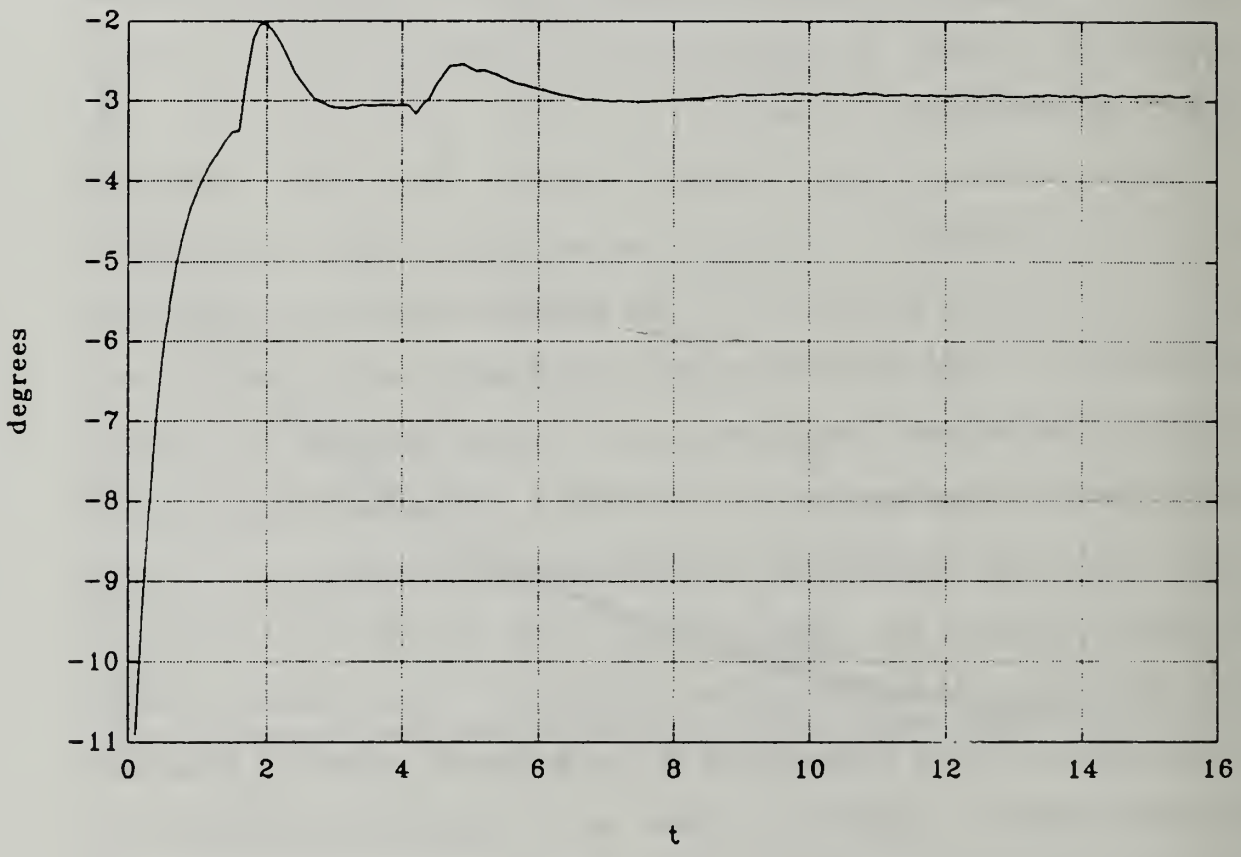

Figure 27. Rudder Time History for the Simulation of Figure 26.

After introducing the previous table of data into the simulation program, a figure eight manoeuvre was performed by the vehicle, Figure 28. The rudder time history of the above manoeuvre was also plotted, Figure 29, The small values of the rudder angle even for a complicated manoeuvre like this prove the advantage of the method, over the use of line segments only.

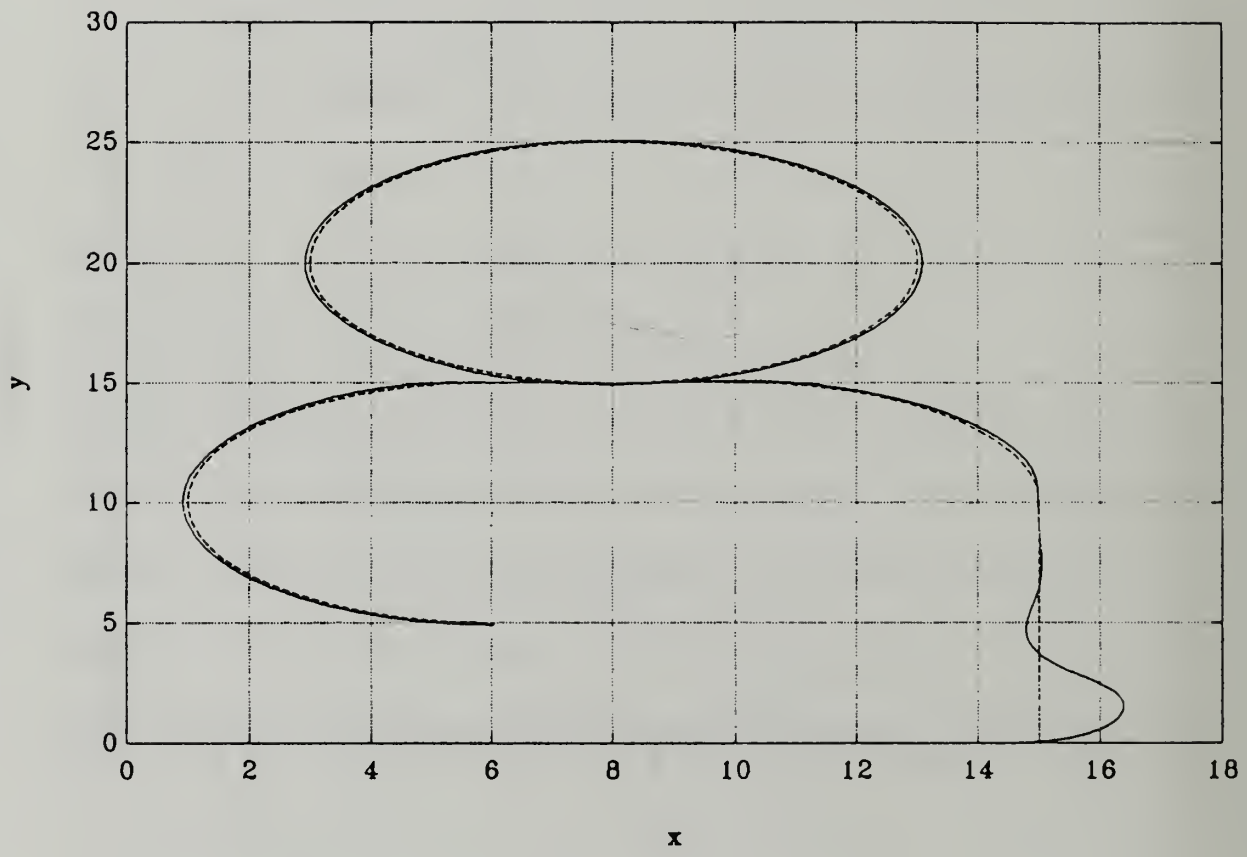

Figure 28. Figure - Eight (8) Manoeuvre.

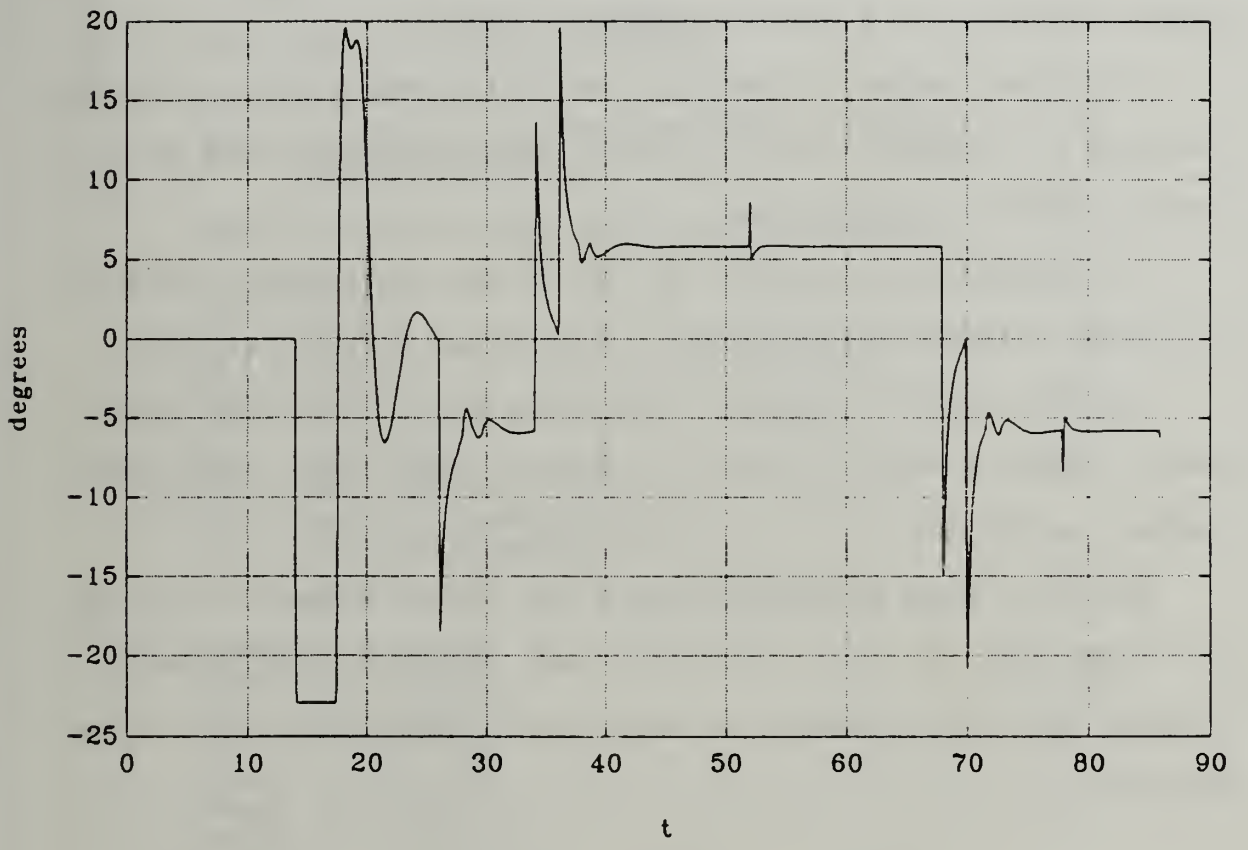

Figure 29. Rudder Time History for the Manoeuvre of Figure 28.

CONCLUSIONS AND RECOMMENDATIONS

The main conclusions and contributions of this work can be summarized as follows :

1. The use of Nomoto's model allows for a very efficient steering control design with excellent stability properties when coupled to a pursuit guidance scheme.

2. An approximate stability condition was developed which is easy to compute and provides a very good estimate of the exact stability condition.

3. Sensitivity analysis of the stability curves revealed the most critical hydrodynamic coefficients for stability.

4. The use of circular reference paths achieves smooth path response along curved segments with very reasonable rudder activity.

Finally, some recommendations for future research include the analysis of path accuracy and disturbance rejection properties of the scheme, as well as the impact of sensor bias and noise.

APPENDIX A

```
C      PROGRAM NOMO.FOR
C
C
C      FOTIS A. PAPOULIAS - DIMITRIOS A. SIMAKIS
C      NAVAL POSTGRADUATE SCHOOL
C      MONTEREY CALIFORNIA
C      JUNE 1992
C
C      CRITICAL STABILITY PROGRAM
C
C
       REAL K1,K2,K3,L,NR,NV,NDRS,NDRB,IZ,MASS,
      &     NRDOT,NVDOT
C
       OPEN (10,FILE='NOMO.RES',STATUS='NEW')
C
       WEIGHT=435.0
       IZ     =45.0
       L      =7.3
       RHO    =1.94
       G      =32.2
       XG     =0.0104
       MASS   =WEIGHT/G
       MASS   =MASS/(0.5*RHO*L**3)
       IZ     =IZ/(0.5*RHO*L**5)
       XG     =XG/L
C
       YRDOT =-0.00000
       YVDOT =-0.03430
       YR     =+0.00000
       YV     =-0.10700
       YDRS   =+0.01241
       YDRB   =+0.01241
       NRDOT =-0.00047
       NVDOT =-0.00000
       NR     =-0.00390
       NV     =-0.00000
       NDRS   =-0.337*0.01241
       NDRB   =+0.283*0.01241
C
       DH   =(IZ-NRDOT)*(MASS-YVDOT)-
      &      (MASS*XG-YRDOT)*(MASS*XG-NVDOT)
       AA11=((IZ-NRDOT)*YV-(MASS*XG-YRDOT)*NV)/DH
       AA12=((IZ-NRDOT)*(-MASS+YR)-
      &      (MASS*XG-YRDOT)*(-MASS*XG+NR))/DH
```

```
      AA21=((MASS-YVDOT)*NV-(MASS*XG-NVDOT)*YV)/DH
      AA22=((MASS-YVDOT)*(-MASS*XG+NR)-
     &     (MASS*XG-NVDOT)*(-MASS+YR))/DH
      BB11=((IZ-NRDOT)*YDRS-(MASS*XG-YRDOT)*NDRS)/DH
      BB12=((IZ-NRDOT)*YDRB-(MASS*XG-YRDOT)*NDRB)/DH
      BB21=((MASS-YVDOT)*NDRS-(MASS*XG-NVDOT)*YDRS)/DH
      BB22=((MASS-YVDOT)*NDRB-(MASS*XG-NVDOT)*YDRB)/DH
C
      WRITE (*,1006)
      READ  (*,*)    RATIO
C
      BB1=BB11+RATIO*BB12
      BB2=BB21+RATIO*BB22
C
      WRITE (*,1001)
      READ  (*,*)    ICON
      GO TO (100,200), ICON
  100 WRITE (*,1002)
      READ  (*,*)    ZMIN,ZMAX,IZETA
      WRITE (*,1003)
      READ  (*,*)    WN
      INCR=IZETA
      GO TO 50
  200 WRITE (*,1004)
      READ  (*,*)    WNMIN,WNMAX,IWN
      WRITE (*,1005)
      READ  (*,*)    ZETA
      INCR=IWN
C
   50 C1=(AA11*AA22-AA21*AA12)*(AA21*BB1-AA11*BB2)
      C2=(AA11+AA22)*(AA21*BB1-AA11*BB2)
     &+BB2*(AA11*AA22-AA21*AA12)
      C3=-(AA21*BB1-AA11*BB2)**2
      A=C1/C2
      B=C3/C2
C
      DO 1 I=1,INCR
C
        IF (ICON.EQ.1) ZETA=ZMIN +(ZMAX -ZMIN )*(I-1)/(INCR-1)
        IF (ICON.EQ.2) WN  =WNMIN+(WNMAX-WNMIN)*(I-1)/(INCR-1)
C
        IF (ICON.EQ.1) OUT=ZETA
        IF (ICON.EQ.2) OUT=WN
C
        K1=-WN*WN/B
        K2=-(A+2.0*ZETA*WN)/B
C
        B1P=-(AA11+AA22)-BB2*K2
        C1P=(AA11*AA22-AA12*AA21)+(BB2*AA11-BB1*AA21)
     &  *K2-BB2*K1
        C2P=-BB1*K1
```

```
        D1P=(AA11*BB2-AA21*BB1)*K1
        D2P=(BB1*AA22-BB2*AA12)*K1-BB2*K1
        E2P=(AA11*BB2-AA21*BB1)*K1
C
        AQ=B1P*C1P*D1P-D1P*D1P
        BQ=B1P*C1P*D2P+B1P*C2P*D1P-B1P*B1P*E2P-2.0*D1P*D2P
        CQ=B1P*C2P*D2P-D2P*D2P
        DET=BQ*BQ-4.0*AQ*CQ
        IF (DET.LT.0.0) GO TO 1
        XD1=(-BQ+SQRT(DET))/(2.0*AQ)
        XD2=(-BQ-SQRT(DET))/(2.0*AQ)
C
        COEF1=BB1*AA22-BB2*AA12-BB2
        COEF2=BB2*AA11-BB1*AA21
C
        VAL1=1.0+COEF1/(XD1*COEF2)
        VAL2=1.0+COEF1/(XD2*COEF2)
        IF ((VAL1.LT.0.0).OR.(XD1.LT.0.0)) XD1=0.0
        IF ((VAL2.LT.0.0).OR.(XD2.LT.0.0)) XD2=0.0
        XD3=1.0/(2.0*ZETA*WN)
        WRITE (10,2001) XD1,XD2,XD3,OUT,K1,K2
    1 CONTINUE
C
 1001 FORMAT (' ENTER 1 : ZETA VARIATION',/,
     &        '       2 : WN   VARIATION')
 1002 FORMAT (' ENTER MIN,MAX, AND INCREMENTS OF ZETA')
 1003 FORMAT (' ENTER WN')
 1004 FORMAT (' ENTER MIN,MAX, AND INCREMENTS OF WN')
 1005 FORMAT (' ENTER ZETA')
 1006 FORMAT (' ENTER BOW/STERN RUDDER RATIO')
 2001 FORMAT (6E15.5)
      END
```

APPENDIX B

```
C      PROGRAM NOMOV.FOR
C
C
C      FOTIS A. PAPOULIAS - DIMITRIOS A. SIMAKIS
C      NAVAL POSTGRADUATE SCHOOL
C      MONTEREY CALIFORNIA
C      JUNE 1992
C
C
C      SENSITIVITY OF HYDRODYNAMIC COEFFICIENTS PROGRAM
C
C
       REAL K1,K2,K3,L,NR,NV,NDRS,NDRB,IZ,MASS,
      &     NRDOT,NVDOT
C
       OPEN (10,FILE='NOMOV.RES',STATUS='NEW')
C
       WEIGHT=435.0
       IZ    =45.0
       L     =7.3
       RHO   =1.94
       G     =32.2
       XG    =0.0104
       MASS  =WEIGHT/G
       MASS  =MASS/(0.5*RHO*L**3)
       IZ    =IZ/(0.5*RHO*L**5)
       XG    =XG/L
C
       YRDOT =-0.00000
       YVDOT =-0.03430
       YR    =+0.00000
       YV    =-0.10700
       YDRS  =+0.01241
       YDRB  =+0.01241
       NRDOT =-0.00047
       NVDOT =-0.00000
       NR    =-0.00390
       NV    =-0.00000
       NDRS  =-0.337*0.01241
       NDRB  =+0.283*0.01241
C
       DH  =(IZ-NRDOT)*(MASS-YVDOT)-
      &     (MASS*XG-YRDOT)*(MASS*XG-NVDOT)
       AA11=((IZ-NRDOT)*YV-(MASS*XG-YRDOT)*NV)/DH
       AA12=((IZ-NRDOT)*(-MASS+YR)-
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     &      (MASS*XG-YRDOT)*(-MASS*XG+NR))/DH
      AA21=((MASS-YVDOT)*NV-(MASS*XG-NVDOT)*YV)/DH
      AA22=((MASS-YVDOT)*(-MASS*XG+NR)-
     &      (MASS*XG-NVDOT)*(-MASS+YR))/DH
      BB11=((IZ-NRDOT)*YDRS-(MASS*XG-YRDOT)*NDRS)/DH
      BB12=((IZ-NRDOT)*YDRB-(MASS*XG-YRDOT)*NDRB)/DH
      BB21=((MASS-YVDOT)*NDRS-(MASS*XG-NVDOT)*YDRS)/DH
      BB22=((MASS-YVDOT)*NDRB-(MASS*XG-NVDOT)*YDRB)/DH
C
      RATIO=-1.0
C
      BB1=BB11+RATIO*BB12
      BB2=BB21+RATIO*BB22
C
      ZETA=0.8
      WN=3.0
C
      C1=(AA11*AA22-AA21*AA12)*(AA21*BB1-AA11*BB2)
      C2=(AA11+AA22)*(AA21*BB1-AA11*BB2)+BB2
     &    *(AA11*AA22-AA21*AA12)
      C3=-(AA21*BB1-AA11*BB2)**2
      A=C1/C2
      B=C3/C2
C
      K1=-WN*WN/B
      K2=-(A+2.0*ZETA*WN)/B
C
         B1P=-(AA11+AA22)-BB2*K2
         C1P=(AA11*AA22-AA12*AA21)+(BB2*AA11-BB1*AA21)
     &       *K2-BB2*K1
         C2P=-BB1*K1
         D1P=(AA11*BB2-AA21*BB1)*K1
         D2P=(BB1*AA22-BB2*AA12)*K1-BB2*K1
         E2P=(AA11*BB2-AA21*BB1)*K1
C
         AQ=B1P*C1P*D1P-D1P*D1P
         BQ=B1P*C1P*D2P+B1P*C2P*D1P-B1P*B1P*E2P-2.0*D1P*D2P
         CQ=B1P*C2P*D2P-D2P*D2P
         DET=BQ*BQ-4.0*AQ*CQ
         IF (DET.LT.0.0) GO TO 1
         XDNOM=(-BQ+SQRT(DET))/(2.0*AQ)
C
      HOLD=YVDOT
C
      DO 1 I=1,101
C
         VARY=0.5+(I-1)/100.0
         YVDOT=0.5*HOLD+HOLD*(I-1)/100.0
C
      DH   =(IZ-NRDOT)*(MASS-YVDOT)-
     &      (MASS*XG-YRDOT)*(MASS*XG-NVDOT)
```

```
      AA11=((IZ-NRDOT)*YV-(MASS*XG-YRDOT)*NV)/DH
      AA12=((IZ-NRDOT)*(-MASS+YR)-
     &     (MASS*XG-YRDOT)*(-MASS*XG+NR))/DH
      AA21=((MASS-YVDOT)*NV-(MASS*XG-NVDOT)*YV)/DH
      AA22=((MASS-YVDOT)*(-MASS*XG+NR)-
     &     (MASS*XG-NVDOT)*(-MASS+YR))/DH
      BB11=((IZ-NRDOT)*YDRS-(MASS*XG-YRDOT)*NDRS)/DH
      BB12=((IZ-NRDOT)*YDRB-(MASS*XG-YRDOT)*NDRB)/DH
      BB21=((MASS-YVDOT)*NDRS-(MASS*XG-NVDOT)*YDRS)/DH
      BB22=((MASS-YVDOT)*NDRB-(MASS*XG-NVDOT)*YDRB)/DH
      BB1=BB11+RATIO*BB12
      BB2=BB21+RATIO*BB22
C
        B1P=-(AA11+AA22)-BB2*K2
        C1P=(AA11*AA22-AA12*AA21)+(BB2*AA11-BB1*AA21)
     &   *K2-BB2*K1
        C2P=-BB1*K1
        D1P=(AA11*BB2-AA21*BB1)*K1
        D2P=(BB1*AA22-BB2*AA12)*K1-BB2*K1
        E2P=(AA11*BB2-AA21*BB1)*K1
C
        AQ=B1P*C1P*D1P-D1P*D1P
        BQ=B1P*C1P*D2P+B1P*C2P*D1P-B1P*B1P*E2P-2.0*D1P*D2P
        CQ=B1P*C2P*D2P-D2P*D2P
        DET=BQ*BQ-4.0*AQ*CQ
        IF (DET.LT.0.0) GO TO 1
        XD1=(-BQ+SQRT(DET))/(2.0*AQ)
        XD2=(-BQ-SQRT(DET))/(2.0*AQ)
C
        COEF1=BB1*AA22-BB2*AA12-BB2
        COEF2=BB2*AA11-BB1*AA21
C
        VAL1=1.0+COEF1/(XD1*COEF2)
        VAL2=1.0+COEF1/(XD2*COEF2)
        IF ((VAL1.LT.0.0).OR.(XD1.LT.0.0)) XD1=0.0
        IF ((VAL2.LT.0.0).OR.(XD2.LT.0.0)) XD2=0.0
        XD3=1.0/(2.0*ZETA*WN)
        WRITE (10,2001) XD1/XDNOM,VARY
    1 CONTINUE
C
 1001 FORMAT (' ENTER 1 : ZETA VARIATION',/,
     &        '       2 : WN   VARIATION')
 1002 FORMAT (' ENTER MIN,MAX, AND INCREMENTS OF ZETA')
 1003 FORMAT (' ENTER WN')
 1004 FORMAT (' ENTER MIN,MAX, AND INCREMENTS OF WN')
 1005 FORMAT (' ENTER ZETA')
 1006 FORMAT (' ENTER BOW/STERN RUDDER RATIO')
 2001 FORMAT (2E15.5)
      END
```

APPENDIX C

```
C     PROGRAM SIMU.FOR
C
C
C     FOTIS A. PAPOULIAS - DIMITRIOS A. SIMAKIS
C     NAVAL POSTGRADUATE SCHOOL
C     MONTEREY CALIFORNIA
C     JUNE 1992
C
C     SIMULATION PROGRAM FOR CONTROL AND GUIDANCE
C     ALONG STRAIGHT LINES AND
C     CIRCULAR TRAJECTORIES
C
C
      REAL K1,K2,L,NR,NV,NDRS,NDRB,IZ,MASS,
     &     NRDOT,NVDOT
C
      OPEN (10,FILE='SIMU.RES',STATUS='NEW')
      OPEN (11,FILE='SIMU.DAT',STATUS='OLD')
C
C     GEOMETRIC PROPERTIES
C
      WEIGHT=435.0
      IZ    =45.0
      L     =7.3
      RHO   =1.94
      G     =32.2
      XG    =0.0104
      MASS  =WEIGHT/G
      MASS  =MASS/(0.5*RHO*L**3)
      IZ    =IZ/(0.5*RHO*L**5)
      XG    =XG/L
      PI    =3.1415926536
C
C     HYDRODYNAMIC COEFFICIENTS
C
      YRDOT =-0.00000
      YVDOT =-0.03430
      YR    =+0.00000
      YV    =-0.10700
      YDRS  =+0.01241
      YDRB  =+0.01241
      NRDOT =-0.00047
      NVDOT =-0.00000
      NR    =-0.00390
      NV    =-0.00000
```

```
      NDRS  =-0.337*0.01241
      NDRB  =+0.283*0.01241
C
      DH  =(IZ-NRDOT)*(MASS-YVDOT)-
     &      (MASS*XG-YRDOT)*(MASS*XG-NVDOT)
      AA11=((IZ-NRDOT)*YV-(MASS*XG-YRDOT)*NV)/DH
      AA12=((IZ-NRDOT)*(-MASS+YR)-
     &      (MASS*XG-YRDOT)*(-MASS*XG+NR))/DH
      AA21=((MASS-YVDOT)*NV-(MASS*XG-NVDOT)*YV)/DH
      AA22=((MASS-YVDOT)*(-MASS*XG+NR)-
     &      (MASS*XG-NVDOT)*(-MASS+YR))/DH
      BB11=((IZ-NRDOT)*YDRS-(MASS*XG-YRDOT)*NDRS)/DH
      BB12=((IZ-NRDOT)*YDRB-(MASS*XG-YRDOT)*NDRB)/DH
      BB21=((MASS-YVDOT)*NDRS-(MASS*XG-NVDOT)*YDRS)/DH
      BB22=((MASS-YVDOT)*NDRB-(MASS*XG-NVDOT)*YDRB)/DH
C
      RATIO=-1
C
      BB1=BB11+RATIO*BB12
      BB2=BB21+RATIO*BB22
C
      ZETA=0.8
      WN=3.0
      DIST=1.0
C
      C1=(AA11*AA22-AA21*AA12)*(AA21*BB1-AA11*BB2)
      C2=(AA11+AA22)*(AA21*BB1-AA11*BB2)+BB2
     &    *(AA11*AA22-AA21*AA12)
      C3=-(AA21*BB1-AA11*BB2)**2
      A=C1/C2
      B=C3/C2
C
      K1=-WN*WN/B
      K2=-(A+2.0*ZETA*WN)/B
C
      STIME=150.0
      DELTAT=0.1
      ITIME=STIME/DELTAT
C
C     INITIAL CONDITIONS
C
      PSI=0.0
      V=0.0
      R=0.0
      Y=0.0
      X=0.0
      X1=0.0
      Y1=0.0
      J=0
C
    PRINT*,'ENTER NUMBER OF POINTS'
```

```
      READ*,NUM
C
C  THE SIMULATION STARTS FROM THE POINT (0.0,0.0) AND THE NEXT
C
C  POINT (X2,Y2) IS PROVIDED BY THE DATA FILE.
C
C
C  ITURN   DETERMINES THE WAY THE VEHICLE IS
C          TO TURN TO PROCEED FOR THE NEXT POINT.
C
C  ITURN 1 FOR CIRCLE
C
C  ITURN 0 FOR DISTANCE
C
C  DD      DETERMINES THE DISTANCE OF THE VEHICLE
C          FROM XZ,Y2 WHEN THE TURN STARTS.
C
C  IPATH   DETERMINES IF THE DESIRE PATH TO X2,Y2
C          IS A CIRCULAR ONE OR A STRAIGHT LINE.
C
C  IPATH 1 FOR CIRCULAR
C
C  IPATH 0 FOR STRAIGHT LINE
C
C  ICCW  0 FOR CW
C        1 FOR CCW
C
C  X0,Y0  IS THE DESIRED CENTER OF THE CIRCULAR PATH
C         AND X0,Y0 ARE PROVIDED FROM THE DATA FILE
C         AND IF NOT THE DATA
C         FILE PROVIDES ZERO VALUES.
C
      READ(11,*) X2,Y2,ITURN,DD,IPATH,ICCW,X0,Y0
C
C     SIMULATION STARTS
C
      DO 1 I=1,ITIME
      TIME=I*DELTAT
C
      X21=X2-X1
      Y21=Y2-Y1
      IF ((X21.EQ.0.).AND.(Y21.GT.0.)) ANA=+0.5*PI
      IF ((X21.EQ.0.).AND.(Y21.LT.0.)) ANA=-0.5*PI
      IF (X21.EQ.0.) GO TO 93
      ANA=ATAN(ABS(Y21)/ABS(X21))
      IF ((X21.GE.0.).AND.(Y21.GE.0.)) ANA=    ANA
      IF ((X21.GE.0.).AND.(Y21.LT.0.)) ANA=   -ANA
      IF ((X21.LT.0.).AND.(Y21.GE.0.)) ANA= PI-ANA
      IF ((X21.LT.0.).AND.(Y21.LT.0.)) ANA=-PI+ANA
 93   CONTINUE
C
```

```
C       EQUATIONS OF MOTION
C
        PSIDOT=R
        VDOT=AA11*V+AA12*R+BB1*DR
        RDOT=AA21*V+AA22*R+BB2*DR
        YDOT=SIN(PSI)+V*COS(PSI)
        XDOT=COS(PSI)-V*SIN(PSI)
C
C       FIRST ORDER INTEGRATION
C
        PSI=PSI+DELTAT*PSIDOT
        V=V+DELTAT*VDOT
        R=R+DELTAT*RDOT
        Y=Y+DELTAT*YDOT
        X=X+DELTAT*XDOT
C
      IF (PSI.GT.(2.0*PI)) PSI=PSI-2.0*PI
C
      YPR=(Y-Y1)*COS(ANA)-(X-X1)*SIN(ANA)
      PSIC=-ATAN(YPR/DIST)
      IF (IPATH.EQ.0) DR=K1*SIN((PSI-PSIC-ANA))+K2*R
C
C
        IF (IPATH.EQ.1) THEN
        DALF1=((X-X0)*(X-X0)+(Y-Y0)*(Y-Y0))**0.5
        RADIUS=((X1-X0)*(X1-X0)+(Y1-Y0)*(Y1-Y0))**0.5
        X3=X0+RADIUS/DALF1*(X-X0)
        Y3=Y0+RADIUS/DALF1*(Y-Y0)
        THONE=PI/2.-DIST/(2.*RADIUS)
C       WRITE (*,*) X,Y,X3,Y3,X0,Y0,RADIUS
C
         IF (ICCW.EQ.0) THEN
C
           IF ((X21.GT.0.0).AND.(Y21.GT.0.0)) THEN
           THTHR=ASIN(ABS(X0-X3)/RADIUS)
           THTWO=PI-THONE-THTHR
           DISTF=2.0*RADIUS*SIN(DIST/(2.0*RADIUS))
           X4=X3+DISTF*SIN(THTWO)
           Y4=Y3+DISTF*COS(THTWO)
           DALF2=((X-X4)*(X-X4)+(Y-Y4)*(Y-Y4))**0.5
           BALFA=SIN(DIST/RADIUS)*DALF1/DALF2
           IF (BALFA.GT.1.0) BALFA=1.0
           IF (BALFA.LE.1.0) AALFA=ASIN(BALFA)
           SIGMA=AALFA-THONE
           THTWO=0.5*PI-THTWO
           END IF
C
           IF ((X21.GT.0.0).AND.(Y21.LT.0.0)) THEN
           THTHR=ASIN(ABS(Y0-Y3)/RADIUS)
           THTWO=PI-THONE-THTHR
           DISTF=2.0*RADIUS*SIN(DIST/(2.0*RADIUS))
```

```
            X4=X3+DISTF*COS(THTWO)
            Y4=Y3-DISTF*SIN(THTWO)
            DALF2=((X-X4)*(X-X4)+(Y-Y4)*(Y-Y4))**0.5
            BALFA=SIN(DIST/RADIUS)*DALF1/DALF2
            IF (BALFA.GT.1.0) BALFA=1.0
            IF (BALFA.LE.1.0) AALFA=ASIN(BALFA)
            SIGMA=AALFA-THONE
            THTWO=-THTWO
            END IF
C
            IF ((X21.LT.0.0).AND.(Y21.LT.0.0)) THEN
            THTHR=ASIN(ABS(X0-X3)/RADIUS)
            THTWO=PI-THONE-THTHR
            DISTF=2.0*RADIUS*SIN(DIST/(2.0*RADIUS))
            X4=X3-DISTF*SIN(THTWO)
            Y4=Y3-DISTF*COS(THTWO)
            DALF2=((X-X4)*(X-X4)+(Y-Y4)*(Y-Y4))**0.5
            BALFA=SIN(DIST/RADIUS)*DALF1/DALF2
            IF (BALFA.GT.1.0) BALFA=1.0
            IF (BALFA.LE.1.0) AALFA=ASIN(BALFA)
            SIGMA=AALFA-THONE
            THTWO=1.5*PI-THTWO
            END IF
C
            IF ((X21.LT.0.0).AND.(Y21.GT.0.0)) THEN
            THTHR=ASIN(ABS(Y0-Y3)/RADIUS)
            THTWO=PI-THONE-THTHR
            DISTF=2.0*RADIUS*SIN(DIST/(2.0*RADIUS))
            X4=X3-DISTF*COS(THTWO)
            Y4=Y3+DISTF*SIN(THTWO)
            DALF2=((X-X4)*(X-X4)+(Y-Y4)*(Y-Y4))**0.5
            BALFA=SIN(DIST/RADIUS)*DALF1/DALF2
            IF (BALFA.GT.1.0) BALFA=1.0
            IF (BALFA.LE.1.0) AALFA=ASIN(BALFA)
            SIGMA=AALFA-THONE
            THTWO=PI-THTWO
            END IF
C
C
         END IF
C
         IF (ICCW.EQ.1) THEN
C
            IF ((X21.GT.0.0).AND.(Y21.GT.0.0)) THEN
            THTHR=ASIN(ABS(Y0-Y3)/RADIUS)
            THTWO=PI-THONE-THTHR
            DISTF=2.0*RADIUS*SIN(DIST/(2.0*RADIUS))
            X4=X3+DISTF*COS(THTWO)
            Y4=Y3+DISTF*SIN(THTWO)
            DALF2=((X-X4)*(X-X4)+(Y-Y4)*(Y-Y4))**0.5
            BALFA=SIN(DIST/RADIUS)*DALF1/DALF2
```

```
            IF (BALFA.GT.1.0) BALFA=1.0
            IF (BALFA.LE.1.0) AALFA=ASIN(BALFA)
            SIGMA=AALFA-THONE
            THTWO=THTWO
            END IF
C
            IF ((X21.GT.0.0).AND.(Y21.LT.0.0)) THEN
            THTHR=ASIN(ABS(X0-X3)/RADIUS)
            THTWO=PI-THONE-THTHR
            DISTF=2.0*RADIUS*SIN(DIST/(2.0*RADIUS))
            X4=X3+DISTF*SIN(THTWO)
            Y4=Y3-DISTF*COS(THTWO)
            DALF2=((X-X4)*(X-X4)+(Y-Y4)*(Y-Y4))**0.5
            BALFA=SIN(DIST/RADIUS)*DALF1/DALF2
            IF (BALFA.GT.1.0) BALFA=1.0
            IF (BALFA.LE.1.0) AALFA=ASIN(BALFA)
            SIGMA=AALFA-THONE
            THTWO=1.5*PI+THTWO
            END IF
C
            IF ((X21.LT.0.0).AND.(Y21.LT.0.0)) THEN
            THTHR=ASIN(ABS(Y0-Y3)/RADIUS)
            THTWO=PI-THONE-THTHR
            DISTF=2.0*RADIUS*SIN(DIST/(2.0*RADIUS))
            X4=X3-DISTF*COS(THTWO)
            Y4=Y3-DISTF*SIN(THTWO)
            DALF2=((X-X4)*(X-X4)+(Y-Y4)*(Y-Y4))**0.5
            BALFA=SIN(DIST/RADIUS)*DALF1/DALF2
            IF (BALFA.GT.1.0) BALFA=1.0
            IF (BALFA.LE.1.0) AALFA=ASIN(BALFA)
            SIGMA=AALFA-THONE
            THTWO=PI+THTWO
            END IF
C
            IF ((X21.LT.0.0).AND.(Y21.GT.0.0)) THEN
            THTHR=ASIN(ABS(X0-X3)/RADIUS)
            THTWO=PI-THONE-THTHR
            DISTF=2.0*RADIUS*SIN(DIST/(2.0*RADIUS))
            X4=X3-DISTF*SIN(THTWO)
            Y4=Y3+DISTF*COS(THTWO)
            DALF2=((X-X4)*(X-X4)+(Y-Y4)*(Y-Y4))**0.5
            BALFA=SIN(DIST/RADIUS)*DALF1/DALF2
            IF (BALFA.GT.1.0) BALFA=1.0
            IF (BALFA.LE.1.0) AALFA=ASIN(BALFA)
            SIGMA=AALFA-THONE
            THTWO=0.5*PI+THTWO
            END IF
C
C
        END IF
C
```

```
       IF (ICCW.EQ.0)  DR=K1*SIN(PSI-THTWO+SIGMA)+K2*R
       IF (ICCW.EQ.1)  DR=K1*SIN(PSI-THTWO-SIGMA)+K2*R
C
      END IF
C
      IF (DR.GT.0.4)  DR=0.4
      IF (DR.LT.-0.4) DR=-0.4
C
      WRITE(10,2001) TIME,X,Y,PSI,PSIC,DR,YPR
C
      TURN=(X-X2)*(X-X2)+(Y-Y2)*(Y-Y2)
      TURNC=SQRT(TURN)
      TURNL=SQRT(TURN-YPR*YPR)
      IF (ITURN.EQ.1) TD=TURNL
      IF (ITURN.EQ.0) TD=TURNC
      IF (TD.LT.DD) THEN
  8       J=J+1
          IF (J.EQ.NUM) GO TO 2002
              IF (J.NE.NUM) THEN
               X1=X2
               Y1=Y2
               READ(11,*) X2,Y2,ITURN,DD,IPATH,ICCW,X0,Y0
               GO TO 1
              END IF
          END IF
C
  1     CONTINUE
 2001 FORMAT (7E15.5)
 2002  STOP
       END
```

```
C     PROGRAM REFERENCE PATH
C
C     DIMITRIOS SIMAKIS
C     NAVAL POSTGRADUATE SCHOOL
C     MONTEREY, CALIFORNIA
C     JUNE 1992
C
      OPEN (11,FILE='SIMUR.REF',STATUS='OLD')
      OPEN (10,FILE='SIMUR.RES',STATUS='NEW')
      X1=0.0
      Y1=0.0
      PI=3.141
      PRINT*,'ENTER NUMBER OF POINTS'
      READ*,NUM
C
      DO 1 I=1,NUM+1
```

```
      IF (I.EQ.1) THEN
      XR=X1
      YR=Y1
      WRITE (10,100) XR,YR
      GO TO 1
      END IF
      READ (11,*) X2,Y2,ITURN,DD,IPATH,ICCW,X0,Y0
      THETA=0.0
      RADIUS=((X1-X0)*(X1-X0)+(Y1-Y0)*(Y1-Y0))**0.5
      X21=X2-X1
      Y21=Y2-Y1
      DO 2 J=1,200
          THETA= THETA+0.01745
          IF (IPATH.EQ.1) THEN
           IF (ICCW.EQ.0) THEN
              IF (X21.GT.0.0.AND.Y21.GT.0.0) THEN
                 XR=X0-RADIUS*COS(THETA)
                 YR=Y0+RADIUS*SIN(THETA)
              END IF
C
              IF (X21.GT.0.0.AND.Y21.LT.0.0)  THEN
                 XR=X0+RADIUS*SIN(THETA)
                 YR=Y0+RADIUS*COS(THETA)
              END IF
C
              IF (X21.LT.0.0.AND.Y21.GT.0.0)  THEN
                 XR=X0-RADIUS*SIN(THETA)
                 YR=Y0-RADIUS*COS(THETA)
              END IF
C
              IF (X21.LT.0.0.AND.Y21.LT.0.0)  THEN
                 XR=X0+RADIUS*COS(THETA)
                 YR=Y0-RADIUS*SIN(THETA)
              END IF
            END IF
C
             IF (ICCW.EQ.1) THEN
                 IF (X21.GT.0.0.AND.Y21.GT.0.0) THEN
                 XR=X0+RADIUS*SIN(THETA)
                 YR=Y0-RADIUS*COS(THETA)
                 END IF
C
              IF (X21.GT.0.0.AND.Y21.LT.0.0)  THEN
                 XR=X0-RADIUS*COS(THETA)
                 YR=Y0-RADIUS*SIN(THETA)
              END IF
C
              IF (X21.LT.0.0.AND.Y21.GT.0.0)  THEN
                 XR=X0+RADIUS*COS(THETA)
                 YR=Y0+RADIUS*SIN(THETA)
              END IF
```

```
C
              IF (X21.LT.0.0.AND.Y21.LT.0.0)  THEN
                 XR=X0-RADIUS*SIN(THETA)
                 YR=Y0+RADIUS*COS(THETA)
              END IF
            END IF
          END IF
C
             IF (IPATH.EQ.0) THEN
               XR=X2
               YR=Y2
            END IF
C
            WRITE (10,100) XR,YR
            DIS=((XR-X2)*(XR-X2)+(YR-Y2)*(YR-Y2))**0.5
            IF (DIS.LT.0.1) THEN
               X1=X2
               Y1=Y2
               GO TO 1
            END IF
  2       CONTINUE
 100     FORMAT (2E15.5)
 1     CONTINUE
         STOP
          END
```

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